# A 3D Model Analysis of Artificial Knee Joint during Passive Deep Flexion 

## by

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#### Abstract

The objective of this study is to analyze kinetics and kinematics of knee prosthesis during deep knee flexion. We constructed a 3D model of total knee joint and performed the simulation analysis during passive knee flexion while the flexion angle was between $40^{\circ}$ and $140^{\circ}$. Although we could introduce the solutions while the flexion angle was between $40^{\circ}$ and $111^{\circ}$, the solution diverged when the flexion angle was over $112^{\circ}$. The problem was caused by the incongruence between patello-femoral surfaces.


Keywords: Deep knee flexion, Total knee prosthesis, A 3D model simulation, Passive flexion, Contact analysis

## 1. Introduction

The objective of this study is to analyze kinetics and kinematics of knee prosthesis using 3D model.

Replacement technique of knee prosthesis has been available for the joint rheumatism and knee osteoarthritis. Most of the currently used artificial knee joints have been designed based on the kinetics and kinematics at level walking. Prosthetic patients have to perform various kinds of lower limb activities besides level walking in daily life. Therefore it is necessary to extend a range of knee motion, thereby improving their quality of life. The prosthesis which should be developed in the next generation is expected to be able to make deep knee flexion. In order to assess the performance of such a new type of prosthesis, we have to use mathematical model since in vivo experiments are not allowed using a newly developed prosthesis.

For the above reason, we have developed a 3D mathematical model of knee joint. Our 3D model made it possible to reproduce comprehensive knee motion, as it included not only the tibio-femoral joint but also the patello-femoral joint. Using the model, we performed the simulation analysis of the passively moved deep knee flexion.

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## 2. Materials and Methods

### 2.1 The 3D model

We constructed a 3D model of total knee joint arthroplasty, referring to the Wismans's model ${ }^{1 \text { 1) }}$ and the Hirokawa's model ${ }^{2), 3)}$ (Fig.1). In our model, the tibio-femoral and patello-femoral compartments were incorporated. The femur was described with the fixed coordinate system and the tibia and the patella with the moving systems. The shapes of prosthetic articular surfaces were described with parametric polynomial equations respectively.


Fig. 1 The 3D mathematical model of a knee joint.
The model included three muscles: quadriceps, hamstrings and gastrocnemius, and three ligaments: medial and lateral collateral ligament and patella tendon. Friction between two surfaces was neglected.

In the model, it was assumed the articular surfaces are rigid and the contact was reduced to contact at a point. Simultaneous contact between tibia and femur at both the medial and the lateral sides was required; same was true between the patella and the femur. When the flexion angle becomes large, the post and cam start to contact. Friction between articulating surfaces and the force of inertia were neglected.

In order to avoid the repetitive explanations for the model equations, we will explain only about the tibio-femoral joint however the following explanations are also applicable to the patello-femoral joint. When tibio-femoral joint contacts on the points respectively, the following relationship holds at each point,

$$
\begin{equation*}
\mathbf{c}=\mathbf{a}+\mathbf{T} \boldsymbol{\delta} \tag{1}
\end{equation*}
$$

where $\mathbf{c}$ is a position vector of a point on the femoral surface, $\boldsymbol{\delta}$ is a position vector of a point
on the tibial surface, and $\mathbf{a}$ is a vector between both the origins of the femoral and the tibial coordinates. Matrix $\mathbf{T}$ is a rotation transform matrix. Since a normal vector $\boldsymbol{n}$ on the femoral surface is orthogonal to a tangential plane of the tibial surface, we obtain

$$
\begin{equation*}
\left(\mathbf{n}, \mathbf{T} \tau_{\mathbf{r}}\right)=0,\left(\mathbf{n}, \mathbf{T} \tau_{\mathbf{s}}\right)=0 \tag{2}
\end{equation*}
$$

Next, we will introduce the force and moment equilibrium equations. Vectors $\mathbf{r}$ and $\boldsymbol{\rho}$ are the position vectors on the femur and the tibia respectively. Vector $\mathbf{v}$ is a vector of pull direction of a ligament. Vector $\mathbf{v}$ can be expressed as,

$$
\begin{equation*}
\mathbf{v}=\mathbf{r}-\mathbf{a}-\mathbf{T} \boldsymbol{\rho} / \sqrt{(\mathbf{r}-\mathbf{a}-\mathbf{T} \boldsymbol{\rho})^{2}} \tag{3}
\end{equation*}
$$

From Eq.(4), the force equilibrium condition is expressed as,

$$
\begin{equation*}
\mathbf{F}_{\mathbf{e}}+\sum p_{i} \mathbf{n}_{i}+\sum f_{j} \mathbf{v}_{j}+\sum F_{m k} \mathbf{v}_{k}=0 \tag{4}
\end{equation*}
$$

where $\mathbf{F}_{\mathbf{e}}$ is the external force including weight of tibia and ground reaction force, $p$ is contact force, $\mathbf{F}_{\mathbf{m}}$ is muscle strength, and $f$ is tensile force of the ligament or the tendon. A ligament or a tendon is assumed to be a set of eight non-linear springs, and $f$ can be expressed as a function of its length $l$. The spring constants was determined referring the literature ${ }^{1), 4)}$.

$$
f= \begin{cases}\sum_{i=1}^{8} k_{i}\left(l_{i}-l_{i 0}\right)^{2} & l>l_{0}  \tag{5}\\ 0 & l \leq l_{0}\end{cases}
$$

The moment equilibrium condition is expressed as,

$$
\begin{equation*}
\mathbf{M}_{\mathbf{e}}+M_{r} \boldsymbol{\lambda}+\sum\left\{p_{i}\left(\mathbf{T}_{i}\right) \times \mathbf{n}_{i}\right\}+\sum\left\{f_{j}\left(\mathbf{T} \boldsymbol{\rho}_{i}\right) \times \mathbf{v}_{j}\right\}+\sum\left\{F_{m k}\left(\mathbf{T} \boldsymbol{\rho}_{k}\right) \mathbf{v}_{k}\right\}=0 \tag{6}
\end{equation*}
$$

where Me is an external moment vector. Scalar $M_{r}$ is a magnitude of required moment to achieve knee flexion angle $\theta$, and vector $\lambda$ is a unit vector to describe the moment axis direction.

We solved the nonlinear simultaneous equation by Newton-Raphson method. Then, we introduced the position/orientation of each component, the positions of contact points, contact force, and other variables.

### 2.2 Input data

We chose the SSF prosthesis (Scorpio SuperFlex, Stryker Co., U.S.A.) for the subject of our simulation. We replace the patella component by the shape of dome referring the catalog of Stryker. Because the post of this model was difficult to describe with the parametric polynomial equation, we replace the post by circular cylinder.

Simulation was performed during passive knee flexion, because it would not generate strong contact forces and would be easily treated. The input data which are necessary for the simulation are three kinds of variables as follows; flexion angle, muscle force and the external force to flex the knee.

We obtained the data of the above three kinds of variables by performing the following in-vitro experiment ${ }^{5}$. The femoral bone of a cadaver knee was fixed on a jig, and each muscle was pulled by a weight; quadriceps: $0.08[\mathrm{kN}]$, hamstrings: $0.02[\mathrm{kN}]$, gastrocnemius: $0[\mathrm{kN}]$ respectively. Then the tibia was carried through from $40^{\circ}-140^{\circ}$ of knee flexion with the external force and we monitored the external force (Fig.2). Though the experiment was performed subject to the knee with a new type of prosthesis, CFK (Complete Flexion Knee) ${ }^{6)}$, we diverted the result to our simulation because the variation of external force was not affected so much depending on the type of prostheses.


Fig. 2 The external force (input data).

## 3. Results

We could calculate while the flexion angle was between $40^{\circ}$ and $111^{\circ}$. But the solution diverged over $112^{\circ}$. When the flexion angle is over $70^{\circ}$, we saw the contact between post and cam. We show the results in Fig. 3 through Fig.6. Figure 3 shows the contact force. Figure 4 through 6 shows the contact points. Figure 7 shows the position of the knee joint when the flexion angle is $111^{\circ}$.
$\checkmark$ med. $\square$ lat. $\longrightarrow$ post-cam
$\triangle-$ med. $-\square$ lat.



Fig. 3 The contact force.


Fig. 4 The contact points of patello-femoral joint.


Fig. 5 The contact points of patello-femoral joint.


Fig. 6 The contact points of tibio-femoral joint.


Fig. 7 The position of the knee joint at $111^{\circ}$.

## 4. Discussion

The simulation results demonstrated that the solution diverged when the flexion angle was over $112^{\circ}$. The reason why the solution diverged over this angle was attributed to the patello-femoral surfaces. When the flexion angle increased, the contact points on the femur came to the medial and lateral edges of condyles. Because the patella was not supposed to contact in this area, the contact loses stability and could not satisfy the condition of point contact. To verify this assumption, we simulated without the patella, setting the tension of patella tendon to the appropriate direction and confirmed that the solution converged. Thus we could conclude that the reason why the solution diverged was due to the irregular contacts between the patellar and the medial and lateral edges of femoral condyles.

We should reshape the patello-femoral surfaces in a reasonable shape in order to introduce the stable solution for the simulation. Also we recommend that, for designing the artificial knee joint which are capable of deep flexion, the shape of patello-femoral joint must be reconsidered. Especially, the femoral surface is important because the contact surface would be large and lap over the tibio-femoral contact surface.

The conventional prostheses cannot flex deeply because of their tibio-femoral articulating structures. And therefore the patients are instructed not to flex their prosthetic knees deeply in order to avoid the possible problems. Consequently the problems with the patello-femoral joint have not been reported so many. Even if a newly designed prosthesis makes it possible to felx the knee deeply due to improvement of the tibio-femoral articulation structure, still the problems will remain with the patello-femoral joint. We can conclude that the shape of patello-femoral joint must be reconsidered to design the artificial knee joint which can flex deeply.

On the other hand, during passive flexion, not only the patello-femoral but also the tibio-femoral joints often lost stability. Our simulation recreated this effect by the negative contact force at lateral condyle in high knee flexion. This does not mean the critical lift off, because this effect occurred just temporary and the magnitude of the negative contact force was very small; We could continue the calculation neglecting this problem and the contact force turned to be positive soon.

## 5. Conclusion

We performed the simulation during passive knee flexion, using our 3D mathematical model. We could introduce the stable solutions as long as the flexion angle was $40^{\circ}-111^{\circ}$, yet we could not when the flexion angle was over $112^{\circ}$. The problem was caused by the incongruence between patello-femoral surfaces. When the flexion angle increased, the contact points on the femur came to the medial and lateral edges of condyles. This phenomenon should be taken into account when designing the prosthesis capable of deep flexion.

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