Memoirs of the Faculty of Engineering, Kyushu University, Vol.68, No.1, March 2008

Proposed Structure of Neutrino Based on Weak-Charge and Weak-Dipole-Moment Interaction

by

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(Received February 4, 2008)

Abstract

The neutrino structure has been studied with three major assumptions for constituent particles: (1) Weak charge is capable of working as a weak dipole moment and the electromagnetic self-energy in the Fermi gauge produces the neutrino mass, (2) motions of neutrino constituent particles are governed in two separate ways by extended Dirac and extended Klein-Gordon equations, and (3) the neutrino system has implicit internal subspaces that give constraints on kinetic motions and potential interactions. So-called vector and axial-vector movements play a role of time and spatial motions, respectively, and generate the individual potential propagations at the same time. The exchange relation for operators of constituent motions explains the creation of half-integer spin as well as a periodical vibration motion.

Keywords: Neutrino, Weak charge, Weak dipole moment, Vibration

1. Introduction

Neutrinos and electrons commonly exist in nature and are classified into the same group of lepton[1]. Neutrinos have a half-integer spin in the restricted direction. They are treated to make the weak interaction of vector(V) and axial-vector(AV) types[2], and supposed to own a quite small mass[3]. Quantum mechanics allots quantized integer angular momentums to particle orbital motions. The half-integer spin seems to come from a sophisticated internal motion of constituent particles. Electroweak theory[4,5] unified the electromagnetic and weak interaction, and explained the mass of field bosons. However, such theory was not applied to the neutrino structure, and the neutrino structure and mass generation were not made clear. It is interesting to consider that a neutrino is

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composed of constituent particles, and they have V- and AV-type motions with some special degrees of freedom, which has not been established so far. In this study, we attempt to explain the origin of the small neutrino mass as well as the half-integer spin in the restricted direction.

For covariant treatment of electromagnetic field, Fermi (Feynman) gauge[6] is established by the use of auxiliary field[7] $\hat{B}^0$. The auxiliary field may take an essential role in producing the neutrino mass. The auxiliary field will be coupled with neutrino constituent motions with bold assumptions. A discussion will be presented on an external auxiliary field. Existence of the external auxiliary field may make a neutrino to be dissociated. This reaction may be useful for engineering purposes of making use of the weak interaction energy in future.

The internal motion of constituents in neutrino system is proposed by making several assumptions. (1) Interaction potentials are generated by weak charge and weak dipole moment in the Fermi gauge, (2) motions of neutrino constituent particles are basically governed by an Dirac and Klein-Gordon equations[2] in an extended manner, and (3) the neutrino system has internal subspaces for kinetic motions which are different from the conventional space. At first, the internal motions are considered to be governed by both V- and AV-potentials, and then the potential matrix types are revised to produce the neutrino mass through the potential interaction.

2. Weak-Electric Dipole Moment as Original Potential Source

In the conventional treatment for particles like atoms, the electromagnetic interaction is caused by the electric charge e that serves as a potential source. The electromagnetic system possesses the spin with up- and down-degrees of freedom in orientation. When such a potential source is simply replaced by a weak-electric charge Q, it is supposed to be impossible to explain the experimental fact that neutrino has the restricted spin orientation.

We postulate that the weak-electric charge Q has the feature of working as a weak-electric-dipole moment Q_d, so that this moment may play an important role in the potential interaction as a counterpart of the charge. The dipole moment Q_d is assumed to have a relationship with the weak-electric charge Q through a specified length. It is natural for the Compton wavelength $\hbar/mc$ to serve as this length: $Q = Q_d mc/\hbar$ where m is the mass of neutrino. The weak-electric dipole moment Q_d works as a magnetic moment $\mu_0 v Q_d$ for a particle moving with velocity v. Thereafter, Q and Q_d are called briefly the electric charge and the dipole moment, respectively. The weak-electric and weak-magnetic fields are also simply designated as the electric and the magnetic ones, respectively.

The basic interaction between Q and Q_d is supposed to take place as illustrated in **Fig. 1**, where two particles having Q and Q_d are moving in the z-direction with a left-rotated spiral motion. The velocity v_z of Q_d generates the magnetic field of dipole type $B(v_z Q_d)$ as shown in the upper side by dashed curves. This magnetic field is expected to be compensated by the field $B(v_\phi Q)$, which is drawn in the lower side by solid curves being created by the rotational velocity v_ϕ of Q. Meanwhile, the circular magnetic field $B(v_\phi Q_d)$ is produced by the velocity v_ϕ of the dipole moment Q_d, and is cancelled by that in the reversed direction $B(v_z Q)$ formed by v_z

of the charge Q. Thus, both the magnetic fields generated by Q_d and Q have individually the opposite directions in principle. The two particles interact with each other to have the reduced magnetic-field energy, so that they may be in an either stable or equilibrium state.

Figure 1 is based on the left-rotated motions, and the velocities of straight motions are in the same direction. This situation is expected to be suited for the basic internal motion of constituent particles. However, the sum of angular momentums gets higher in this case. We need some mechanism for reducing the total angular momentum: A mass-polarity configuration will be introduced later, which includes particles of negative mass states.

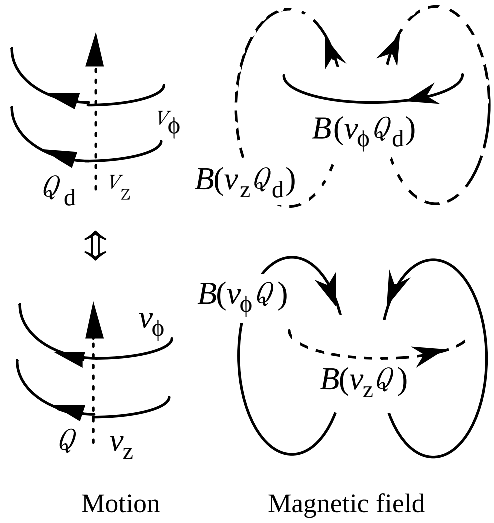

Fig. 1 Magnetic field generated by particles with electric charge Q and dipole moment Q_d. The two particles are traveling in the z-direction with a left-rotated spiral motion.

3. Reduced Mass, Extended Dirac Equation and Related Definitions

In this study, covariant quantities[2] are expressed by variables with use of the imaginary unit i. Variables with subscript indicate actual real values. Covariant properties are expressed by superscripted variables, which include the imaginary unit i in some cases.

The Dirac equation utilizes four gamma matrices of γ_ν with ν =0~3.[6] A neutrino is assumed to consist of four types of constituent particles, according to the number of the basic gamma matrices. It is postulated, at first, that the constituent particles exist in either real or imaginary mass states. The mass of internal particle in the imaginary mass state is expressed by im_ν^{int} in terms of a real value m_ν^{int}. Accordingly, the square of imaginary mass is expressed by $(im_\nu^{\mathrm{int}})^2 = s_\nu^m (m_\nu^{\mathrm{int}})^2$ with $s_\nu^m = -1$, while that of real mass by $(m_\nu^{\mathrm{int}})^2 = s_\nu^m (m_\nu^{\mathrm{int}})^2$ with $s_\nu^m = +1$. The total mass m_s of neutrino is taken to be positive, and is written by $(m_s)^2 = s_s^m |m_s|^2$ with $s_s^m = +1$. The total mass m_s is defined by motions with internal masses as

$$m_s = \sqrt{\sum_\nu s_\nu^m \sum_\mu \left(m_\nu^{int} \dot{x}_\nu^\mu\right)^2}\ , \tag{1}$$

where the velocity $\dot{x}_\nu^\mu$ of constituent particle ν is defined classically as $d/d(c\tau/i)$ by the real value of intrinsic time τ of the system. Eq. (1) gives a classical momentum of

$$q_{cc,\nu}^{int,\mu} = \frac{\partial m_s}{\partial \dot{x}_\nu^\mu} = s_\nu^m \left(m_\nu^{int}\right)^2 \dot{x}_\nu^\mu \left\{\sum_\nu s_\nu^m \sum_\mu \left(m_\nu^{int} \dot{x}_\nu^\mu\right)^2\right\}^{-1/2} = \frac{1}{m_s} s_\nu^m \left(m_\nu^{int}\right)^2 \dot{x}_\nu^\mu = \frac{1}{\sqrt{s_\nu^m}\,|m_s|} s_\nu^m \left(m_\nu^{int}\right)^2 \dot{x}_\nu^\mu\ ,$$

$$\sqrt{s_\nu^m}\, q_{cc,\nu}^{int,\mu} = s_\nu^m \left\{\left(m_\nu^{int}\right)^2 / |m_s|\right\} \dot{x}_\nu^\mu\ . \tag{2}$$

The bracket in the left-hand side in eq. (2) contains no imaginary value: Adoption of a reduced mass of $(m_\nu^{int})^2/|m_s|$ with the polarity s_ν^m enables us to use the real quantity of equivalent mass for the momentum. It is useful to reconstruct eq. (1) in a linear form, taking the equivalent mass into account. The assumption of the relativistic relationship of $\Sigma(\dot{x}_\nu^\mu)^2 = 1$ changes the square of eq. (1) into

$$(m_s)^2 = \sum_\nu s_\nu^m \sqrt{|m_s|^2 \sum_\mu \frac{\left(m_\nu^{int}\right)^4}{|m_s|^2} \left(\dot{x}_\nu^\mu\right)^2} \sqrt{\sum_\mu \left(\dot{x}_\nu^\mu\right)^2} = |m_s| \sum_\nu s_\nu^m \sqrt{\sum_\mu \frac{\left(m_\nu^{int}\right)^4}{|m_s|^2} \left(\dot{x}_\nu^\mu\right)^2}$$

$$= |m_s| \sum_\nu s_\nu^m \sqrt{\sum_\mu (m_\nu)^2 \left(\dot{x}_\nu^\mu\right)^2}\ , \quad \text{with } m_\nu = (m_\nu^{int})^2/|m_s|\ ,$$

where m_ν becomes positive. Division of this equation by $|m_s|$ gives

$$m_t \equiv (m_s)^2/|m_s| = s_s^m |m_s| = \sum_\nu s_\nu^m \sqrt{\sum_\mu (m_\nu)^2 \left(\dot{x}_\nu^\mu\right)^2} = \sum_{\nu=0\sim 3} s_\nu^m m_\nu\ . \tag{3}$$

Thus, the original real and imaginary properties of constituent masses are written by the linear sum of real-value m_ν with $s_\nu^m = \pm 1$: The form of the linear summation allows us to treat the original internal motions as the external movements in terms of the reduced mass m_ν. Thereafter, we take this view and simply call $s_\nu^m m_\nu$ positive or negative mass states, which may be suited to the treatment by the Dirac equation.

We use the definition of gamma matrices[2)] as

$$\gamma_0 = \begin{pmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{pmatrix},\quad \gamma_k = \begin{pmatrix} 0 & -\sigma_k \\ \sigma_k & 0 \end{pmatrix},\quad \gamma_5 = \begin{pmatrix} 0 & i\sigma_0 \\ i\sigma_0 & 0 \end{pmatrix},\quad k = 1,2,3\ , \tag{4}$$

where the Pauli's spin matrices are

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The unit matrix 1 is given by

$\begin{pmatrix} \sigma_0 & 0 \\ 0 & \sigma_0 \end{pmatrix}$ with $\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

The following Dirac-based equation with inclusion of AV-momentum is one of candidates for expressing the motion of constituent particle ν:

$$\left\{ \sum_{\mu=0\sim3} \left(\gamma_\mu p_{\mu,\nu V} + \gamma_5 \gamma_\mu p_{\mu,\nu A} \right) \right\} \psi_\nu = m_\nu \psi_\nu \,, \tag{5}$$

where $p_{\mu,\nu V}$ and $p_{\mu,\nu A}$ stand for V- and AV-momentums. The V-type 4-momentum $p_{\mu,\nu V}$ possesses the matrix property of γ_μ, and accordingly $x_{\mu,\nu V}$ has γ_μ. We, at first, treat as if the neutrino constituent mass m_ν has the matrix of time component, i.e. γ_0. Multiplication of eq. (5) by γ_0 leads to

$$\left\{ \sum_{\mu=0\sim3} \left(\gamma_0 \gamma_\mu p_{\mu,\nu V} + \gamma_0 \gamma_5 \gamma_\mu p_{\mu,\nu A} \right) \right\} \psi_\nu = \gamma_0 \gamma_0 m_\nu \psi_\nu = m_\nu \psi_\nu \,, \tag{6}$$

The view of eq. (6) assigns the unit matrix 1 to the time variables through $\gamma_0 \gamma_0 p_{0,\nu V} = p_{0,\nu V}$ and $\gamma_0 \gamma_0 x_{0,\nu V} = x_{0,\nu V}$. The unit matrix offers a flexible function as equivalent time component, as explained in section 5. In spite of addition of γ_0, we still call $p_{\mu,\nu V}$ and $p_{\mu,\nu A}$ V- and AV-momentums in this study.

Defining superscripted values of

$$\gamma^\mu = \gamma_\mu \text{ for } \mu = 0 \sim 3, \text{ and } \gamma^5 = \gamma_5 / i$$

makes $\gamma^0 \gamma^\mu$ and $\gamma^0 \gamma^5 \gamma^\mu$ the Hermite matrix, and gives

$$\left(\gamma^0 \gamma^\mu \right)^2 = 1, \quad \left(\gamma^0 \gamma^5 \gamma^\mu \right)^2 = 1 .$$

The relation

$$\gamma_0 \gamma_\mu p_{\mu,\nu V} = \gamma^0 \gamma^\mu p^\mu_{\nu V}, \quad \gamma_0 (\gamma_5 / i) \gamma_\mu i p_{\mu,\nu A} = \gamma^0 \gamma^5 \gamma^\mu p^\mu_{\nu A}$$

suggests us to take

$$p^\mu_{\nu V} = p_{\mu,\nu V}, \; p^\mu_{\nu A} = i p_{\mu,\nu A}$$

for $\mu=$0~3. Properties of variables are summarized in **Table 1**. It is noted that both the position variables $x^\mu_{\nu V}$ and $x^\mu_{\nu A}$ for the V- and AV-motions have the same matrix of $\gamma^0 \gamma^\mu$ with V-type, differing those of momentum and potentials. This is required for the AV-field such as $B^k_{\nu A}$ to possess γ^5 to retain AV-properties. The time components of V- and AV-potentials have reversed complex type. This comes from the derivation of the potentials as explained in section 5.

The motion of many-body system is usually described in the cm frame. We, therefore, consider that particle ν with Q_d in eq. (6) make their motion on a reference position. This reference position is supplied by the motion of a basic particle, which is designated as *b*. The situation of **Fig. 1** is suitable to this view, when particle ν with Q_d resides on the motion of particle *b* with charge Q. However, this situation produces a large angular momentum of the system. It is expected for sum of angular momentums to be reduced by introduction of flexible common positions, which may have the property coming from a nature of boson as described bellow.

Table 1 Super- and sub-scripted variables and matrix properties. The quantity $\hat{B}^0$ stands for auxiliary field, which will be explained in section 5. For potential treatment, time coordinate x^0 stands for that for potential propagation $x^{0\,fd}$ as described in section 5.

V motion	AV motion
$\begin{pmatrix} x^0_{\nu V} \\ x^k_{\nu V} \end{pmatrix} = \begin{pmatrix} x_{0,\nu V}/i \\ x_{k,\nu V}/i \end{pmatrix}, \quad \gamma^0\gamma^\mu$ $\begin{pmatrix} p^0_{\nu V} \\ p^k_{\nu V} \end{pmatrix} = \begin{pmatrix} p_{0,\nu V} \\ p_{k,\nu V} \end{pmatrix}, \quad \gamma^0\gamma^\mu$	$\begin{pmatrix} x^0_{\nu A} \\ x^k_{\nu A} \end{pmatrix} = \begin{pmatrix} x_{0,\nu A} \\ x_{k,\nu A} \end{pmatrix}, \quad \gamma^0\gamma^\mu$ $\begin{pmatrix} p^0_{\nu A} \\ p^k_{\nu A} \end{pmatrix} = \begin{pmatrix} ip_{0,\nu A} \\ ip_{k,\nu A} \end{pmatrix}, \quad \gamma^0\gamma^5\gamma^\mu$
$\begin{pmatrix} A^0_{\nu V} \\ A^k_{\nu V} \end{pmatrix} = \begin{pmatrix} iA_{0,\nu V} \\ A_{k,\nu V} \end{pmatrix}, \quad \gamma^0\gamma^\mu$ $\hat{B}^0 = -\Sigma\partial A^\mu/\partial x^\mu, \quad 1$	$\begin{pmatrix} A^0_{\nu A} \\ A^k_{\nu A} \end{pmatrix} = \begin{pmatrix} A_{0,\nu A} \\ iA_{k,\nu A} \end{pmatrix}, \quad \gamma^0\gamma^5\gamma^\mu$ $\hat{B}^0 = -\Sigma\partial A^\mu/\partial x^\mu, \quad -\gamma^5$

The velocities of particle b should satisfy the relation as $\Sigma(\dot{x}_b^\mu)^2 = 1$ as inferred form eq. (1). For example, the V-velocity has the matrix property as

$$\{\dot{x}^\mu_{bV}\} = \left(\dot{x}^0_{bV},\, \dot{x}^1_{bV},\, \dot{x}^2_{bV},\, \dot{x}^3_{bV}\right)^t \Leftrightarrow \{\gamma^0\gamma^\mu\} = \left(\gamma^0\gamma^0,\, \gamma^0\gamma^1,\, \gamma^0\gamma^2,\, \gamma^0\gamma^3\right)^t, \tag{7}$$

where { } indicates a representation of vector. We assume the relationship between the velocity and the matrix is not always fixed and it can be changed with holding $\Sigma(\dot{x}_b^\mu)^2 = 1$. This feature may be admitted for boson, where matrices always work as a squared form as in the Klein-Gordon equation. Then, the matrix property is considered possible to be the transformed by means of a transformation matrix U_ν as

$$\{\gamma^0\gamma^{\mu'}\} = U_\nu\{\gamma^0\gamma^\mu\},$$

where the unitary property is set as $U^t_\nu U_\nu = 1$. Simple candidates for the transformation matrix are

$$U_\nu = \begin{cases} \gamma_\nu & \text{for } \nu = 0, 1, 3 \\ \gamma_\nu / i & \text{for } \nu = 2 \end{cases}.$$

Conversion of position and momentum by matrices U_ν of $\nu = 0, 2$ makes the z-component of angular momentum inverted, and that by U_3 changes it to zero in a classical consideration. In contrast, transformation by matrix U_1 holds the value of the angular momentum in the z-direction. Unlike matrices U_ν of $\nu = 0, 2, 3$, U_1 may be unsuited for the transformation matrix. Then, we treat particle 1 to own electric charge $Q_b = Q_d m_b c / \hbar$, and to serve as the boson in the conventional space to produce the basic common motion. Quantities related to particle 1 itself are often specified by the use of 1b thereafter. In contrast, other particles $\nu = 0, 2, 3$ possess the dipole moment Q_d. When the matrix-property vector $\{\gamma^0 \gamma^{\mu'}\}$ is rearranged into the standard order of eq. (7), the position vector is expressed in U^ν by

$$\left\{x'^{\mu'}_{bV}\right\}_{U^\nu} = U^t_\nu \left\{x^{\mu}_{bV}\right\}_{U^c} \text{ or } x'^{\mu'}_{bV} = u^{t,\mu'\mu}_\nu x^{\mu}_{bV} = u^{\mu\mu'}_\nu x^{\mu}_{bV} , \tag{8}$$

where the variable x in U^c is denoted by x' in U^ν after transformation, and the matrix element $u^{\mu\mu'}_\nu$ is either +1 or –1. The unitary property of $U^t_\nu U_\nu = 1$ keeps the relation of $\Sigma(\dot{x}^{\mu}_{bV})^2 = \Sigma(\dot{x}^{\mu'}_{bV})^2$ after the transformation. We regard the status of $\{x^{\mu}_{bV}\}$ as the motion in conventional space U^c and $\{x'^{\mu}_{bV}\}$ as that in subspace U^ν. The common motion is also set for AV-movement in the same way as for V-type.

The situation of transformation is illustrated in **Fig. 2**, where the drawing is made, for example, for $\nu = 0$. The particle 1b goes in the z-direction with right-rotated motion as seen by dashed lines in (a). The dashed lines in (b) show the motion of particle 1*b* transferred by matrix U_0, while the solid ones indicate the movement of particle $\nu = 0$ relative to the dashed ones. The situation in (b) is viewed in U^c as (c). The angular momentums in (a) and (b) are summed into a large value. Reducing the summed value requires a different mechanism. This is accomplished by existence of both internal negative-mass state and AV-motion, as explained in later sections. The potential propagation takes place through massless photons. We assume that the photons fly in the same space of U^c without receiving the conversion U_ν that is defined for neutrino constituents. Then, the potential interaction is postulated to take place in U^c: (b) is transformed into (c) and interacts with (a) to be consistent with **Fig. 1**.

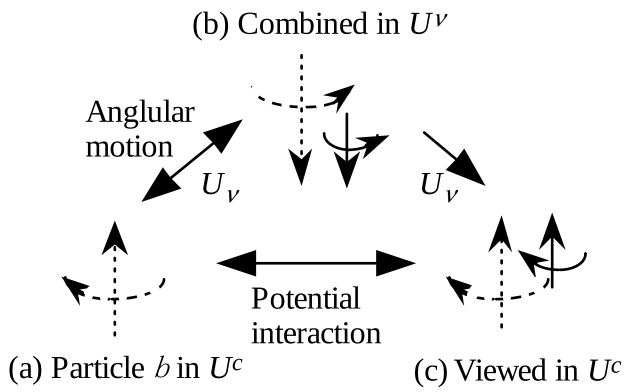

Fig. 2 Example of basic and relative motions for $\nu = 0$. Inset (a) indicates the common motion of particle 1b, and other dashed lines in (b) and (c) express the movement that is originated by the common motion. (b) stands for the sum of converted-common motion and relative movement of particle $\nu = 0$. (c) shows the motion transform from (b) by matrix U_ν. Potential interaction is made between (a) and (c), whereas the particle momentums as well as angular momentums are summed in (a) and (b).

4. Mass Terms of Individual Particles on the Potential-Free Condition

For derivation of individual masses, potentials are dropped in this section. At first, the mass terms are obtained in U^ν, but they are finally expressed by the use of variables in U^c. Particles $\nu = 0$, 2 and 3 have positions $x'^{\mu}_{\nu V} = x'^{\mu}_{\nu a V} + u^{\mu'\mu}_{\nu} x^{\mu'}_{bV}$ for the V-motion in U^ν, where The quantities in U^ν are denoted by attaching dash in such a way as $x'^{\mu}_{\nu V}$. The particle positions are treated to override on that of particle 1*b* with having the same mass polarity. The classical momentum for the V- and AV-motion is written by

$$m_\nu \dot{x}'^{\mu}_{\nu X} = m_\nu \left(\dot{x}'^{\mu}_{\nu a X} + u^{\mu'\mu}_{\nu} \dot{x}^{\mu'}_{bX} \right) = q'^{\mu}_{\nu a X} + u^{\mu'\mu}_{\nu} q^{\mu'}_{bX} \ , \quad X = V \text{ or } A \, .$$

Only for the AV potential-interaction of particle ν, internal mass phases of $\zeta_{\nu bA} = \pi / 2$ or π will be introduced for the velocity of $u^{\mu'\mu}_{\nu} \dot{x}^{\mu'}_{bA}$ in section 5, to facilitate either mass formation for particles 0 and 3 or a flexible *Q*-type potential interaction for particle 2. However, they are not considered here, since potential interaction is not treated at present.

The equation for particles $\nu = 0$, 2 and 3 is thus written by the Dirac-like equation as

$$\left[\sum_{\mu=0\sim3} \left(\gamma^0 \gamma^\mu\right)\left(q'^{\mu}_{\nu a V} + u^{\mu'\mu}_{\nu} q^{\mu'}_{bV}\right) + \sum_{\mu=0\sim3} \left(\gamma^0 \gamma^5 \gamma^\mu\right)\left(q'^{\mu}_{\nu a A} + u^{\mu'\mu}_{\nu} q^{\mu'}_{bA}\right) \right] \psi'_\nu = \lambda'_\nu \psi'_\nu \ \text{ in } \ U^\nu \ , \qquad (9)$$

$$s^m_\nu m_\nu = s^m_\nu \left(m^{\text{int}}_\nu\right)^2 / m^{\text{int}}_s = \lambda'_\nu \ .$$

The linear equation is based on the calculation with 4×4 matrices, and solved as the eigen-value problem on the matrices. Use of the definition of gamma matrices in eq. (4) reduces the equation to 2×2 representation as

$$\begin{pmatrix} X_{11} & X_{21} \\ X_{21} & X_{11} \end{pmatrix}\begin{pmatrix} \alpha_\nu \\ \beta_\nu \end{pmatrix} = \lambda'_\nu \begin{pmatrix} \alpha_\nu \\ \beta_\nu \end{pmatrix},$$

where

$$X_{11} = f'^{0}_{\nu V}\sigma_0 + \sum_{k=1\sim 3} f'^{k}_{\nu A}\sigma_k\,,\quad X_{21} = -f'^{0}_{\nu A}\sigma_0 - \sum_{k=1\sim 3} f'^{k}_{\nu V}\sigma_k\,,$$

$$f'^{\mu}_{\nu V} = q'^{\mu}_{\nu aV} + u^{\mu'\mu}_{\nu} q^{\mu'}_{bV}\,,\quad f'^{\mu}_{\nu A} = q'^{\mu}_{\nu aA} + u^{\mu'\mu}_{\nu} q^{\mu'}_{bA}\,, \tag{10}$$

The eigen values $\lambda'_{\nu\pm}$ is expressed by introduction of a parameter $h=\pm 1$ as

$$\lambda'_{\nu h} = X_{11} + hX_{21} = \left(f'^{0}_{\nu V} - hf'^{0}_{\nu A}\right)\sigma_0 + \sum_{k=1\sim 3}\left(f'^{k}_{\nu A} - hf'^{k}_{\nu V}\right)\sigma_k\,, \tag{11}$$

and its squared one as

$$\left(\lambda'_{\nu h}\right)^2 = \left(f'^{0}_{\nu V} - hf'^{0}_{\nu A}\right)^2 + \sum_{k=1\sim 3}\left(f'^{k}_{\nu A} - hf'^{k}_{\nu V}\right)^2 + 2\left(f'^{0}_{\nu V} - hf'^{0}_{\nu A}\right)\sum_{k=1\sim 3}\left(f'^{k}_{\nu A} - hf'^{k}_{\nu V}\right)\sigma_k\,. \tag{12}$$

The eigen value $\lambda'_{\nu h}$ is accompanied by a 4-compoment eigen vector as

$$\begin{pmatrix} \alpha_\nu \\ \beta_\nu \end{pmatrix}_{h=+} = N_h \begin{pmatrix} -X_{11} + \lambda'_{\nu h} \\ X_{21} \end{pmatrix},\quad \begin{pmatrix} \alpha_\nu \\ \beta_\nu \end{pmatrix}_{h=-} = N_h \begin{pmatrix} X_{21} \\ -X_{11} + \lambda'_{\nu h} \end{pmatrix},$$

where N_h is a normalization factor. Substitution of eq. (11) into the above leads to the form of

$$\begin{pmatrix} \alpha_\nu \\ \beta_\nu \end{pmatrix}_{h=+} = NX_{21}\begin{pmatrix} 1 \\ 1 \end{pmatrix},\quad \begin{pmatrix} \alpha_\nu \\ \beta_\nu \end{pmatrix}_{h=-} = NX_{21}\begin{pmatrix} 1 \\ -1 \end{pmatrix}. \tag{13}$$

The states of $h=\pm 1$ are related to the eigen vectors on angular momentum in Section 6, where $h=-1$ is chosen. In accordance with the angular momentum case, $h=-1$ is adopted in eq. (13).

The squared mass in eq. (12) is assumed to have the unit matrix type. This gives a boundary condition of

$$f'^{0}_{\nu V} - hf'^{0}_{\nu A} = \left(q'^{0}_{\nu aV} + u^{\kappa_\nu 0}_{\nu} q^{\kappa_\nu}_{bV}\right) - h\left(q'^{0}_{\nu aA} + u^{\kappa_\nu 0}_{\nu} q^{\kappa_\nu}_{bA}\right) = 0\,, \tag{14}$$

where $\kappa_\nu = 0,3,2$ for particles $\nu = 0,2,3$, respectively, due to the definition of U_ν. One can see that the complex-type of $q'^{0}_{\nu aV} + u^{\kappa_\nu 0}_{\nu} q^{\kappa_\nu}_{bV}$ mismatches with that of $q'^{0}_{\nu aA} + u^{\kappa_\nu 0}_{\nu} q^{\kappa_\nu}_{bA}$. The AV momentum of $q^{\kappa_\nu, rev}_{\nu A} = u^{\kappa_\nu 0}_{\nu} q'^{\kappa_\nu}_{\nu A} = \left(u^{\kappa_\nu 0}_{\nu} q'^{0}_{\nu aA} + q^{\kappa_\nu}_{bA}\right) = h\left(u^{\kappa_\nu 0}_{\nu} q'^{0}_{\nu aV} + q^{\kappa_\nu}_{bV}\right)$ is treated as the reversed complex type. This complex type is also applied to the potential interaction. However, it is assumed that the following squared-form with normal AV complex type is admitted to be usable for the kinetic momentum in wave function:

$$\left(u^{\kappa_\nu 0}_{\nu} q'^{0}_{\nu aA} + q^{\kappa_\nu}_{bA}\right)^2 + \left(u^{\kappa_\nu 0}_{\nu} q'^{0}_{\nu aV} + q^{\kappa_\nu}_{bV}\right)^2 = 0. \tag{15}$$

The formulation was made in U^{ν} in the above. However, the derivation in U^{c} should give the same mass, although the motion direction may change into different one in U^{c}. The definition corresponding to eq. (10) in U^{ν} is written in U^{c} as

$$f_{\nu V}^{\mu} = u_{\nu}^{\mu\mu'} q'^{\mu'}_{\nu aV} + q_{bV}^{\mu}, \quad f_{\nu A}^{\mu} = u_{\nu}^{\mu\mu'} q'^{\mu'}_{\nu aA} + q_{bA}^{\mu}, \tag{16}$$

The momentum in the time direction should also disappear in U^{c}, in the same way as in eq. (14) in U^{ν}:

$$f_{\nu V}^{0} - h f_{\nu A}^{0} = \left(u_{\nu}^{0\kappa_\nu} q'^{\kappa_\nu}_{\nu aV} + q_{bV}^{0}\right) - h\left(u_{\nu}^{0\kappa_\nu} q'^{\kappa_\nu}_{\nu aA} + q_{bA}^{0}\right) = 0 . \tag{17}$$

We admit that the V-type momentum $q_{bV}^{0,rev} = u_{\nu}^{0\kappa_\nu} q'^{\kappa_\nu}_{\nu aV} + q_{bV}^{0} = h\left(u_{\nu}^{0\kappa_\nu} q'^{\kappa_\nu}_{\nu aA} + q_{bA}^{0}\right)$ works as revered complex type, including potential interaction. For kinetic momentum for wave function, similarly,

$$\left(q'^{\kappa_\nu}_{\nu aV} + u_{\nu}^{0\kappa_\nu} q_{bV}^{0}\right)^2 + \left(q'^{\kappa_\nu}_{\nu aA} + u_{\nu}^{0\kappa_\nu} q_{bA}^{0}\right)^2 = 0 \tag{18}$$

is assumed to hold good with V-type normal momentum. Since the Dirac-like equation gives only the first order relation, there may be uncertainty in the treatment of squared value. Satisfaction of eqs. (14) and (18) produces

$$\lambda'_{\nu h} = \pm\sqrt{\sum_{k=1\sim 3}\left(f'^{k}_{\nu A} - h f'^{k}_{\nu V}\right)^2} = \pm\sqrt{\sum_{k\neq\kappa_\nu}\left(q'^{k}_{\nu aV} + u_{\nu}^{k'k} q_{bV}^{k'}\right)^2 + \sum_{k\neq\kappa_\nu}\left(q'^{k}_{\nu aA} + u_{\nu}^{k'k} q_{bA}^{k'}\right)^2} \text{ in } U^{\nu}, \tag{19}$$

and that of (17) and (15) leads to

$$\lambda_{\nu h} = \pm\sqrt{\sum_{k=1\sim 3}\left(f_{\nu A}^{k} - h f_{\nu V}^{k}\right)^2} = \pm\sqrt{\sum_{k\neq\kappa_\nu}\left(u_{\nu}^{kk'} q'^{k'}_{\nu aV} + q_{bV}^{k}\right)^2 + \sum_{k\neq\kappa_\nu}\left(u_{\nu}^{kk'} q'^{k'}_{\nu aA} + q_{bA}^{k}\right)^2} \text{ in } U^{c}, \tag{20}$$

The momentums in the κ_ν direction, thus, disappear in eqs. (19) and (20). In these equations, no explicit terms are retained on the motions in the time direction. The quantities in eqs. (19) and (20) have either plus or minus value, and are called positive and negative mass states. The positive state is defined when the direction of physical velocity is identical to that of momentum, and the negative state vise versa. One can see that the Dirac-like equation produces the mass with real value, corresponding to the linearized situation of eq. (3).

The complex property allocation of $q'^{k}_{\nu V} = q'_{k,\nu V}$ and $q'^{k}_{\nu A} = i q'_{k,\nu A}$ suggests that whole the V-motion should serve as the time for the AV-one: the V-motion takes a role of time-movement for the AV-one on the basis of the complex type. When $\lambda'_{\nu\pm} \approx 0$, the kinetic energies of V- and AV-motions give the magnitude of 1: 1.

Thereafter, we take the view in U^{c} that particle ν follows the Dirac-like equation in U^{c} as

$$\left[\sum_{\mu=0\sim 3}\left(\gamma^{0}\gamma^{\mu}\right)\left(q_{\nu aV}^{\mu} + q_{bV}^{\mu}\right) + \sum_{\mu=0\sim 3}\left(\gamma^{0}\gamma^{5}\gamma^{\mu}\right)\left(q_{\nu aA}^{\mu} + q_{bA}^{\mu}\right)\right]\psi_{\nu} = \lambda_{\nu}\psi_{\nu} , \tag{21}$$

$$\lambda_{\nu h} = \pm\sqrt{\sum_{\mu=0\sim 3}\left(q_{\nu aV}^{\mu} + q_{bV}^{\mu}\right)^2 + \sum_{\mu=0\sim 3}\left(q_{\nu aA}^{\mu} + q_{bA}^{\mu}\right)^2} , \tag{22}$$

with constraints of eqs. (15) and (18).

As stated in the description on **Fig. 2**, the neutrino system is expected to include both positive and negative mass states. We consider that particle 1b has the flexible feature and is regarded as the boson type. The particle is treated to follow the Klein-Gordon-type equation with the V- and AV-motions. We consider that the particle is composed of two motions with internal positive- and negative-mass polarity $s_b^\pi = \pm 1$, for ease of reduction of total angular momentum. The equation for particle 1b may be written by the extension of Klein-Gordon one in U^c as

$$\left[\sum_{\pi=\pm} s_b^\pi \left(\sum_{\mu=0\sim3} \left(\gamma^0\gamma^\mu\right)^2 \left(q_{1bV\pi}^\mu\right)^2 + \sum_{\mu=0\sim3} \left(\gamma^0\gamma^5\gamma^\mu\right)^2 \left(q_{1bA\pi}^\mu\right)^2 \right)\right]\psi_{1b} = \left(\lambda_{1b}\right)^2 \psi_{1b} \ ,$$

where $s_b^{\pi=+} = +1$ and $s_b^{\pi=-} = -1$. In accordance with the complex type in eqs. (14) and (17), we set the reversed complex-type as $q_{1bV\pi}^{0,rev}$ and $q_{1bA\pi}^{3,rev}$ for potential interaction. As indicated later, the reversed complex-type momentum makes much milder contribution to the constituent motion than normal-type one. The reversed complex-type motion is expected to have a minimum degree of freedom. For normal momentum for wave function, therefore, we additionally assume the constraint between $\pi = \pm$ as

$$\left(q_{1bX+}^0\right)^2 = \left(q_{1bX-}^0\right)^2, \quad \left(q_{1bX+}^3\right)^2 = \left(q_{1bX-}^3\right)^2, \ \ X = V \text{ or } A \, , \tag{23}$$

$$q_{1bV\pi}^{0,rev} = q_{1bV\pi}^3, \quad q_{1bA\pi}^{3,rev} = q_{1bA\pi}^0 \ .$$

Use of $(\gamma^0\gamma^k)^2 = 1$ and $(\gamma^0\gamma^5\gamma^k)^2 = 1$ leads to

$$\lambda_{1b} = \sqrt{\sum_{\pi=\pm} s_b^\pi \left(\sum_{\mu=0\sim3} \left(q_{1bV\pi}^\mu\right)^2 + \sum_{\mu=0\sim3} \left(q_{1bA\pi}^\mu\right)^2 \right)} \ , \tag{24}$$

with constraints of eq. (23). The value λ_{1b} is taken to be positive for the neutrino system. The mass terms of eqs. (22) and (24) constitute Lagrangian and subsequently Hamiltonian, by the use of potential interaction terms described in the next section.

5. Potential Generation and Propagation

We consider that the V- and AV-potentials propagate through the flight of V- and AV-photons. The potentials are considered to travel in U^c: All positions and velocities (or momentums) are expressed in U^c for the potential generation and propagation. Suppose that two particles ρ and σ exist at the same intrinsic time τ, where ρ takes a particle type in $1b+, 1b-, 0, 2$ or 3, and σ also stands for one in those. When the potential propagates from the particle σ at position $\left(x_{\sigma V}^\mu, x_{\sigma A}^\mu\right)$ to that of ρ at $\left(x_{\rho V}^\mu, x_{\rho A}^\mu\right)$, the square of the difference of positions is written at the same τ by

$$\left(x_{\rho V}^0 - x_{\sigma V}^0\right)^2 + \sum_{k=1\sim3} \left(x_{\rho V}^k - x_{\sigma V}^k\right)^2 + \left(x_{\rho A}^0 - x_{\sigma A}^0\right)^2 + \sum_{k=1\sim3} \left(x_{\rho A}^k - x_{\sigma A}^k\right)^2 \ ,$$

where positions are denoted in a summed form as inferred from Eq. (22) in U^c. By the use of either retarded or advanced time $\tau' = \tau + \varepsilon$, potentials arrive at the position x_ρ at τ under the condition of

$$\left(x^0_{\rho V} - x'^0_{\sigma V}\right)^2 + \sum_{k=1\sim 3}\left(x^k_{\rho V} - x'^k_{\sigma V}\right)^2 + \left(x^0_{\rho A} - x'^0_{\sigma A}\right)^2 + \sum_{k=1\sim 3}\left(x^k_{\rho A} - x'^k_{\sigma A}\right)^2$$
$$\equiv \left(x^0_{\rho V} - x'^0_{\sigma V}\right)^2 + \left(\boldsymbol{x}_{\rho V} - \boldsymbol{x}'_{\sigma V}\right)^2 + \left(x^0_{\rho A} - x'^0_{\sigma A}\right)^2 + \left(\boldsymbol{x}_{\rho A} - \boldsymbol{x}'_{\sigma A}\right)^2 = 0 \ .$$

Introduction of a spatial distance d_X and a time one x_X^{0fd} with use of $X = V$ or A changes the equation into

$$d_X \equiv \sqrt{\left(\boldsymbol{x}_{\rho X} - \boldsymbol{x}'_{\sigma X^c}\right)^2} = x_X^{0\,fd} \quad , \tag{25}$$

where

$$x_X^{0\,fd} \equiv \sqrt{-\left(x^0_{\rho V} - x'^0_{\sigma V}\right)^2 - \left(x^0_{\rho A} - x'^0_{\sigma A}\right)^2 - \left(\boldsymbol{x}_{\rho X^c} - \boldsymbol{x}'_{\sigma X^c}\right)^2} \quad ,$$

$$X^c = \begin{cases} A & \text{for } X = V \\ V & \text{for } X = A \end{cases} .$$

The time distance $x_X^{0\,fd}$ is defined in incremental form as

$$\partial x_X^{0\,fd} = -\left(d_X^0\right)^{-1}\left[\left(x^0_{\rho V} - x'^0_{\sigma V}\right)\partial x^0_{\rho V} + \left(x^0_{\rho A} - x'^0_{\sigma A}\right)\partial x^0_{\rho A} + \left(\boldsymbol{x}_{\rho X^c} - \boldsymbol{x}'_{\sigma X^c}\right)\bullet \partial \boldsymbol{x}_{\rho X^c}\right]. \tag{26}$$

We follow the matrix-type definition in **Table 1**. For example, momentum $p^\mu_{\nu X}$, where $X = V$ or A, owns the matrix property of $\gamma^0\gamma^\mu$ for the V-type and $\gamma^0\gamma^5\gamma^\mu$ for AV-type. We assume that the weak charge of neutrino equals the charge e of electromagnetic interaction, and adopt the dipole moment $Q_d = e\hbar/|m_t|$ of neutrino and the charge $Q_\nu = Q_d m_\nu/\hbar$ $= e m_\nu/|m_t|$ of particle ν. In this study, the polarities of Q_d and Q_ν are always taken to be fixed, and their variation in function is considered by means of apparent velocity $\beta^\mu_{\nu X} = p^\mu_{\nu X}/m_\nu$, where $m_\nu > 0$. The apparent velocity contains the mass information such as $s_\nu^m e^{i\zeta_{\nu bA}}$. The moment and charge have the unit matrix. The matrix type of A^μ_X to be generated is treated as either $\gamma^0\gamma^\mu$ or $\gamma^0\gamma^5\gamma^\mu$ in accordance with the matrix property of velocity.

The Lagrangian density $\tilde{L}_X$ is constructed at a certain intrinsic time τ. The density $\tilde{L}_X$ is considered to have basically the same form for *X*=*V* and *A*. There is a difference in potential treatment between sources of Q_d and Q_b. Nevertheless, the basic part of the Lagrangian density $\tilde{L}_{basX}$ is common for potential sources of Q_d and Q_b:

$$\tilde{L}_{basX} = \frac{1}{2\mu_0}\left(\sum_{k=1\sim 3}\frac{E^k_X}{c}\frac{E^k_X}{c} + \sum_{k=1-3}B^k_X B^k_X - \hat{B}^0_X\hat{B}^0_X\right) - \frac{1}{\mu_0}\hat{B}^0_X\sum_{\mu=0\sim 3}\frac{\partial A^\mu_X}{\partial x^\mu_{\nu X}} ,$$

where simple subscripts are utilized here: $\partial x^\mu_{\nu X}$ indicates $\partial x^k_{\nu X}$ and $\partial x^0_{\nu X}$ corresponds to $\partial x^{0\,fd}_{\nu X}$ in the above. We adopted the Fermi gauge for introduction of the auxiliary field $\hat{B}^0$ in the above Lagrangian density. The auxiliary field $\hat{B}^0$ acquires the meaning of the electric/magnetic-like field in the time direction. The potential generated by dipole moment Q_d is described by the Lagrangian density as

$$
\begin{aligned}
&\tilde{L}_{Q_d X} = \tilde{L}_{basX} - V_{Q_d X} \\
&V_{Q_d X} = Q_{dX}\rho_{\nu X} c\hat{\beta}^0_{\nu X} B^0_X + Q_{dX}\rho_{\nu X} c\sum_{k=1\sim 3}\beta^k_{\nu X} B^k_X \,, \\
&Q_{dV} = Q_d / i \,, \quad Q_{dA} = Q_d \,, \\
&\hat{\beta}^0_{\nu V} = \beta^0_{\nu V} = 0 \,, \quad \beta^k_{\nu V} = s^m_\nu \left(\dot{x}^k_{\nu a V} + \dot{x}^k_{bV}\right), \\
&\hat{\beta}^0_{\nu A} = \tilde{\beta}^0_{\nu A} \,, \quad \beta^0_{\nu A} = 0, \quad \beta^k_{\nu A} = s^m_\nu \left(1 - \delta_{k\kappa_\nu}\right)\dot{x}^k_{\nu a A} \,, \\
&\tilde{\beta}^0_{\nu A} = \sqrt{\left(\dot{x}^{\kappa_\nu}_{\nu a A} + \dot{x}^{\kappa_\nu}_{bA}\right)^2 + \chi^{rev}_{\nu b A}\sum_{k\neq\kappa_\nu}\left(\dot{x}^k_{bA}\right)^2}\,, \quad \chi^{rev}_{\nu b A} = \begin{cases} 0 & \text{for } e^{i\zeta_{\nu b A}} = -1 \\ 1 & \text{for } e^{i\zeta_{\nu b A}} = i \end{cases},
\end{aligned}
\tag{27}
$$

where $\rho_{\nu X}$ stands for the spatial particle density. The symbolic expression of velocity is written as

$$
\beta^k_{\nu V} = s^m_\nu q^k_{\nu X} / m_\nu c = s^m_\nu \dot{x}^k_{\nu X} \,,
$$

The velocity $\beta^k_{\nu X}$ is treated to include such mass property as mass polarity and phase $\zeta_{\nu b A}$. We discriminate $\beta^k_{\nu X}$ of regular complex type from $\beta^{k,rev}_{\nu X}$ having reversed complex type. When $\beta^{k,rev}_{\nu X}$ appears, it is root-squared-summed to change the matrix type into unit one, and accordingly makes an extra time velocity $\tilde{\beta}^0_{\nu X}$ in eq. (27). For the V-type, $\hat{\beta}^0_{\nu X}$ has no regular-complex-type value, and is incapable of producing potential in the time direction. It is noted that $\hat{\beta}^0_{\nu X}$ in the *AV*-type retains the matrix of γ^5 as an exceptional treatment. We consider that the root-squared-sum operation deletes only the matrix properties of $\gamma^0\gamma^k$ with $(\gamma^0\gamma^k)^2 = 1$ in original matrix properties of $\gamma^0\gamma^5\gamma^k$ in $\beta^{k,rev}_{\nu A}$ and then γ^5 remains outside the root.

The conventional procedure on partial differentiation of the Lagrangian density produces the auxiliary field as

$$
\hat{B}^0_X = -\sum_{\mu=0\sim 3}\frac{\partial A^\mu_{\nu X}}{\partial x^\mu_{\nu X}} - \mu_0 Q_{dX}\rho_{\nu X} c\hat{\beta}^0_{\nu X} \tag{28}
$$

Use of this relation simplifies the potential propagation equation. The time-component of 4-vector potential is given by

$$
\Box_X A^0_X = -\mu_0 \frac{\partial}{\partial x^0_{\nu X}}\left(Q_{dX}\rho_{\nu X} c\hat{\beta}^0_{\nu X}\right), \tag{29}
$$

where $x^0_{\nu X}$ in this context corresponds to $x^{0fd}_{\nu Xs}$ in the potential propagation. The spatial parts are written as

$$
\Box_X \boldsymbol{A}_X = -\mu_0 \nabla_X \times \left(Q_{dX}\rho_{\nu X} c\boldsymbol{\beta}_{\nu X}\right) - \mu_0 \nabla_X \left(Q_{dX}\rho_{\nu X} c\hat{\beta}^0_{\nu X}\right), \tag{30}
$$

where d’Alembertian is defined with position variables $x^\mu_{\nu X}$ as

$$\Box_X = \sum_{\mu=0\sim3} \frac{\partial^2}{\partial\left(x_{\nu X}^{\mu}\right)^2} ,$$

and the rotation $\nabla_X \times$ and gradient ∇_X operators also work with variables of $x_{\nu X}^{k}$.

Through the function of magnetic moment, the spatial velocity $\boldsymbol{\beta}_{\nu X}$ in eq. (30) generates the vector potential, which may be denoted by $A_X^{k,reg}$. Besides this component, the velocity $\hat{\beta}_{\nu X}^{0}$ in eqs. (29) and (30) produces some potential component, i.e., $\hat{A}_X^{\mu}$ in A_X^{μ} with description of $A_X^{\mu} = A_X^{\mu,reg} + \hat{A}_X^{\mu}$. When a scalar function $\hat{F}_X^{0}$ is adopted, $\hat{A}_X^{\mu}$ is expressed by

$$\hat{A}_X^{\mu} = -\frac{\partial}{\partial x_{\nu X}^{\mu}} \hat{F}_X^{0} ,$$

$$\Box_X \hat{F}_X^{0} = \mu_0 Q_{dX} \rho_\nu c \hat{\beta}_{\nu X}^{0} . \tag{31}$$

Since $\nabla_X \times (\nabla_X \hat{F}_X^{0}) = 0$, it gives $\nabla_X \times \hat{\boldsymbol{A}}_X = \hat{\boldsymbol{B}}_X = 0$: $\hat{\beta}_{\nu X}^{0}$ generates no magnetic field at all.

In addition, introduction of $\boldsymbol{F}_X^{reg}$ in eq. (30) makes the calculation straight-forward as

$$\begin{aligned} \boldsymbol{A}_X^{reg} &= \nabla \times F_X^{reg} , \\ \Box_X \boldsymbol{F}_X^{reg} &= -\mu_0 Q_{dX} \rho_\nu c \boldsymbol{\beta}_{\nu X} . \end{aligned} \tag{32}$$

The charge-type potential generation is written for X=V or A as

$$\begin{aligned} &\tilde{L}_{QX} = \tilde{L}_{basX} - V_{QX} , \\ &V_{QX} = Q_{dX} \rho_{\nu X} c \hat{\beta}_{\nu X}^{0} \hat{B}_X^{0} + Q_\nu \rho_{\nu X} c \sum_{k=1\sim3} \beta_{\nu X}^{k} A_X^{k} , \\ &\hat{\beta}_{\nu V}^{0} = \beta_{\nu V}^{0} = 0 , \quad \beta_{\nu V}^{k} = \begin{cases} s_{1b}^{\pi} \dot{x}_{1bV\pi}^{k} \\ s_2^{m} \left(\dot{x}_{2aV}^{k} + \dot{x}_{bV}^{k} \right) \end{cases} , \\ &\hat{\beta}_{\nu A}^{0} = \tilde{\beta}_{\nu A}^{0} , \ \beta_{\nu A}^{k} = \begin{cases} s_{1b}^{\pi} \beta_{1A\pi}^{k} \\ s_2^{m} (1 - \delta_{k3}) \left(\chi_{2bA}^{nor} \dot{x}_{bA}^{k} \right) \end{cases} , \ \chi_{2bA}^{nor} = -1 , \\ &\tilde{\beta}_{\nu A}^{0} = \begin{cases} \sqrt{\left(\dot{x}_{1bA\pi}^{3} \right)^2} \\ \sqrt{\left(\dot{x}_{2aA}^{3} + \dot{x}_{bA}^{3} \right)^2} \end{cases} , \end{aligned} \tag{33}$$

where $\hat{\beta}_{\nu V}^{0}$ is set at zero for V-motion. The auxiliary field and 4-vector potential are given by

$$\hat{B}_X^{0} = -\sum_{\mu=0\sim3} \frac{\partial A_X^{\mu}}{\partial x_{\nu X}^{\mu}} - \mu_0 Q_{dX} \rho_\nu c \beta_{\nu X}^{0} ,$$

$$\begin{cases} \Box_X A_X^0 = -\mu_0 \dfrac{\partial}{\partial x_{\nu X}^0}\left(Q_{dX}\rho_\nu c\beta_{\nu X}^0\right), \\ \Box_X A_X^k = -\mu_0 c\beta_{\nu X}^k Q_{\nu X}\rho_\nu - \mu_0 \dfrac{\partial}{\partial x_{\nu X}^k}\left(Q_{dX}\rho_\nu c\beta_{\nu X}^0\right). \end{cases} \tag{34}$$

The particle-1b V-motion makes $A_{bV}^0 = 0$. Unlike other particles having Q_d, particle 1b with charge Q_b produces the auxiliary field $\hat{B}^0$ in both V- and AV-types.

The potential calculation needs to take the relativistic effect into account in Eq. (25). Variation of the effective-time increment $\partial x_{\rho Xs}^{0fd}$ at the arrival position produces the change in the potential generation position on the basis of Eq. (25) as

$$\left(d_X\right)^{-1}\left(\boldsymbol{x}_{\rho X} - \dot{\boldsymbol{x}}_{\sigma X}\right)\bullet\left(-\partial\boldsymbol{x}'_{\sigma X}\right) = \partial x_{\rho X}^{0fd} - \partial x'^{0fd}_{\sigma X} .$$

The relativistic time ratio $\partial x'^{0fd}_{\sigma X} / \partial x_{\rho X}^{0fd}$ needs to be factored in potential propagation calculation.

The Hamiltonian density $\tilde{H}_{em}$ for generation of the weak electromagnetic field is derived from the Lagrangian density. The canonical conjugate momentums are expressed by

$$\pi_X^0 = \frac{\partial L}{\partial\left(\partial A_X^0 / \partial x_X^0\right)} = -\frac{\hat{B}_X^0}{\mu_0}, \quad \pi_X^k = \frac{\partial L}{\partial\left(\partial A_X^k / \partial x_X^0\right)} = \frac{1}{\mu_0}\frac{E_X^k}{c} = \frac{1}{\mu_0}\left(-\frac{\partial A_X^0}{\partial x_X^k} + \frac{\partial A_X^k}{\partial x_X^0}\right),$$

where $X = V \text{ or } A$. These give the Hamiltonian density for dipole-moment and charge types as

$$\begin{aligned} \tilde{H}_{em,X} &= \pi_X^0 \frac{\partial A_X^0}{\partial x_X^0} + \sum_{k=1\sim3} \pi_X^k \frac{\partial A_X^k}{\partial x_X^0} - \tilde{L}_{em,X} \\ &= \frac{1}{2\mu_0}\left\{\frac{\boldsymbol{E}}{c}\bullet\left(\frac{\boldsymbol{E}}{c} + 2\nabla A_X^0\right) - \boldsymbol{B}_X\bullet\boldsymbol{B}_X - \hat{B}_X^0\bullet\left(\hat{B}_X^0 + 2\frac{\partial A_X^0}{\partial x_X^0}\right)\right\} + \begin{cases} Q_{dX}\rho_X c\boldsymbol{\beta}_X\bullet\boldsymbol{B}_X \\ Q_\nu\rho_X c\boldsymbol{\beta}_X\bullet\boldsymbol{A}_X \end{cases}, \end{aligned} \tag{35}$$

where the Hamiltonian is composed of the field-energy-density and interaction-energy -density terms. The first one indicates the self-field energy density and is considered to serve as the mass-energy density of the particle motion.

6. Angular Momentum Operator and Neutrino Spin

The angular momentum operator consists of V- and AV-type ones. We first look into the eigen-value and -state for the angular momentum operator in the z-direction in a simple case, where the motion is written with combined coordinates such as $x_X^\mu = x_{\nu aX}^\mu + x_{bX}^\mu$. The angular momentum operator with its matrix type is expressed for a single constituent particle by

$$\begin{aligned} l^3 &= \left\{\left(\gamma^0\gamma^1 x_V^1\right)\left(\gamma^0\gamma^2 p_V^2\right) + \left(\gamma^0\gamma^2 x_V^2\right)\left(\gamma^0\gamma^1 p_V^1\right)\right\} + \left\{\left(\gamma^0\gamma^1 x_A^1\right)\left(\gamma^0\gamma^5\gamma^2 p_A^2\right) + \left(\gamma^0\gamma^2 x_A^2\right)\left(\gamma^0\gamma^5\gamma^1 p_A^1\right)\right\} \\ &= -\gamma^1\gamma^2\left(x_V^1 p_V^2 - x_V^2 p_V^1\right) - \gamma^1\gamma^5\gamma^2\left(x_A^1 p_A^2 - x_A^2 p_A^1\right) = i\sigma_3\begin{pmatrix} l_V^3 & -l_A^3 \\ -l_A^3 & l_V^3 \end{pmatrix}, \end{aligned}$$

$$l_V^3 = x_V^1 p_V^2 - x_V^2 p_V^1, \quad l_A^3 = x_A^1 p_A^2 - x_A^2 p_A^1 .$$

The eigen value $\lambda_{l\pm}$ and eigen function $\xi_\pm$ for the matrix form are

$$\lambda_{l\pm} = i\sigma_3\left(l_V^3 \pm \left(-l_A^3\right)\right) \quad \xi_+ = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \quad \xi_- = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}. \tag{36}$$

The eigen functions $\xi_\pm$ have the same form as in eq. (13), with the parameter $h = \pm 1$. Therefore, it is possible for the eigen function to give eigen values for operators of both the angular momentum and the previous Dirac-like equation of eq. (21). We prefer the negative value of $h = -1$ to the positive one, since the negative one stands for simple sum of angular momentum in the *z*-direction as $\lambda_{l-} = i\sigma_3(l_V^3 + l_A^3)$.

We consider the commutator for the angular momentum and the linear mass operator in the space of U^c. When eq. (11) is converted in U^c and the spatial parts are summed for particles 0, 2 and 3, it gives the linear operator for mass as

$$\lambda_{L-,sp} = \sum_{\substack{k=1\sim3\\ \nu=0,2,3}} \left(\tilde{\partial}_{\nu V}^k + \tilde{\partial}_{\nu A}^k\right)\sigma_k = \sum_{k=1\sim3} \left(\tilde{\partial}_{Vs}^k + \tilde{\partial}_{As}^k\right)\sigma_k , \tag{37}$$

where $\tilde{\partial}_{\nu V}^k$ and $\tilde{\partial}_{\nu A}^k$ stand for the operators for canonical conjugate momentums, and the summation is taken for particles 0, 2 and 3 as $\tilde{\partial}_{Vs}^k$ and $\tilde{\partial}_{As}^k$. The total angular momentum in the direction *k* (=1,2,3) is given in the form of eigen value by

$$\lambda_{ls-}^k = i\sigma_k\left(l_{Vs}^k + l_{As}^k\right), \quad \boldsymbol{l}_{Vs} = \sum_{\nu=0,2,3} \boldsymbol{x}_{\nu V} \times \tilde{\boldsymbol{p}}_{\nu V} \ , \quad \boldsymbol{l}_{As} = \sum_{\nu=0,2,3} \boldsymbol{x}_{\nu A} \times \tilde{\boldsymbol{p}}_{\nu A} \ , \tag{38}$$

where $\tilde{p}_{\nu X}^k = \tilde{\partial}_{\nu X}^k$ indicate the canonical conjugate momentums. The definition produces the commutator of

$$\begin{aligned} &\lambda_{L-,sp}\lambda_{ls-}^3 - \lambda_{ls-}^3\lambda_{L-,sp} \\ &= 2i\sum_{k=1,2}\left(\tilde{\partial}_{Vs}^k + \tilde{\partial}_{As}^k\right)\sigma_k\sigma_3\left(l_{Vs}^3 + l_{As}^3\right) - i\sigma_1\sigma_3\left(\tilde{\partial}_{Vs}^2 + \tilde{\partial}_{As}^2\right) + i\sigma_2\sigma_3\left(\tilde{\partial}_{Vs}^1 + \tilde{\partial}_{As}^1\right), \end{aligned} \tag{39}$$

The wave function of the system requires the condition for the sum of the right-side terms to be zero. Substitution of $l_{Vs}^3 + l_{As}^3 = i(L_3/2)$ and comparison of the coefficients on spin matrices convert the condition into a simple form of

$$\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}\tilde{p}_{Vs}^1 + \tilde{p}_{As}^1\\ \tilde{p}_{Vs}^2 + \tilde{p}_{As}^2\end{pmatrix} - L_3\begin{pmatrix}\tilde{p}_{Vs}^1 + \tilde{p}_{As}^1\\ \tilde{p}_{Vs}^2 + \tilde{p}_{As}^2\end{pmatrix} = 0 . \tag{40}$$

This constitutes an eigen value equation again. The eigen value is obtained to be $L_3 = \pm 1$, that is,

$$l_{Vs}^3 + l_{As}^3 = \pm(1/2)i . \tag{41}$$

The V-type angular motion takes naturally the velocity sum like $\dot{x}_{\nu aV}^k + \dot{x}_{bV}^k$ in the potential interaction, and makes the interaction in the whole directions of $k = 1 \sim 3$. In contrast, the AV-type angular motion retains the flexibility of potential interaction through $\dot{x}_{\nu aA}^k + e^{i\zeta_{\nu bA}}\dot{x}_{bA}^k$. The Q - Q_d interaction system should be based on the left-rotation motion. This suggests that the

V-motion leads to $l_{Vs}^3 = 0$ and AV-one produces $l_{Vs}^3 = -(1/2)i$, to make the negative helicity as $L_3 = -1$.

The eigen value requires for the relation between eigen vectors to be $\tilde{p}_{Vs}^2 + \tilde{p}_{As}^2 = iL_3\left(\tilde{p}_{Vs}^1 + \tilde{p}_{As}^1\right)$ in the Cartesian coordinate. This relation in $L_3 = -1$ leads to

$$\begin{pmatrix} \tilde{p}_{Vs}^1 \\ \tilde{p}_{Vs}^2 \end{pmatrix} = \begin{pmatrix} \cos\Phi & -\sin\Phi \\ \sin\Phi & \cos\Phi \end{pmatrix}_{\Phi=\pi/2} \begin{pmatrix} \tilde{p}_{As}^1 / i \\ \tilde{p}_{As}^2 / i \end{pmatrix},$$

that is, the vector $(\tilde{p}_{Vs}^1, \tilde{p}_{Vs}^2)$ is $\pi/2$ in advance in rotation in comparison to $(\tilde{p}_{As}^1 / i, \tilde{p}_{As}^2 / i)$, and it is written by $\tilde{p}_{Vs}^\phi = \tilde{p}_{As}^\phi / i$ with neglecting the phase. We directly see the relation of eq. (40) in the cylindrical coordinate. A conversion matrix

$$D = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

is defined between momentum vectors in the cylindrical and Cartesian coordinates. Use of D produces

$$D^t \begin{pmatrix} -L_3 & -i & 0 \\ i & -L_3 & 0 \\ 0 & 0 & 0 \end{pmatrix} D = \begin{pmatrix} -L_3 & -i & 0 \\ i & -L_3 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The condition on eq. (40), therefore, is viewed by the cylindrical momentums as

$$\sum_{\substack{\nu=0,2,3 \\ X=V,A}} \begin{pmatrix} -L_3 & -i & 0 \\ i & -L_3 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \sin\theta_{\nu X} \tilde{\partial}_{\nu X}^r \\ r_{\nu X}^{-1} \sin^{-1}\theta_{\nu X} \tilde{\partial}_{\nu X}^\phi \\ \tilde{p}_{\nu X}^3 \end{pmatrix} = \sum_{\substack{\nu=0,2,3 \\ X=V,A}} \begin{pmatrix} -L_3 \sin\theta_{\nu X} \tilde{\partial}_{\nu X}^r - i r_{\nu X}^{-1} \sin^{-1}\theta_{\nu X} \tilde{\partial}_{\nu X}^\phi \\ i\sin\theta_{\nu X} \tilde{\partial}_{\nu X}^r - L_3 r_{\nu X}^{-1} \sin^{-1}\theta_{\nu X} \tilde{\partial}_{\nu X}^\phi \\ 0 \end{pmatrix} = 0 .$$

The condition becomes to

$$\sum_{\substack{\nu=0,2,3 \\ X=V,A}} \left(L_3 \sin\theta_{\nu X} \tilde{\partial}_{\nu X}^r + i r_{\nu X}^{-1} \sin^{-1}\theta_{\nu X} \tilde{\partial}_{\nu X}^\phi\right) = 0 . \tag{42}$$

When the complex types of canonical conjugate momentums are taken into account for V- and AV-motions, real and imaginary components of Eq. (42) lead to the forms of

$$\sum_\nu \sin\theta_{\nu V} \tilde{p}_{\nu V}^r = L_3 \sum_\nu r_{\nu A}^{-1} \sin^{-1}\theta_{\nu A} l_{\nu A}^3 / i = L_3 \tilde{p}_{As}^\phi / i , \tag{43}$$

$$\sum_\nu \sin\theta_{\nu A} \tilde{p}_{\nu A}^r / i = -L_3 \sum_\nu i r_{\nu V}^{-1} \sin^{-1}\theta_{\nu V} l_{\nu V}^3 / i = -L_3 \tilde{p}_{Vs}^\phi . \tag{44}$$

It is plausible for the negative-helicity state of $L_3 = -1$ to produce $\tilde{p}_{As}^\phi / i < 0$ and to make a positive value of the right-hand side of eq. (43). The V-type radial momentums, therefore, have the positive polarity as a whole. This situation is expected to hold good regardless of $L_3 = \pm 1$. Since $\tilde{p}_{Vs}^\phi = \tilde{p}_{As}^\phi / i$, in turn, the AV-type radial momentums take a negative value in eq. (44). The canonical-conjugate radial momentums of $\tilde{p}_{\nu V}^r$ and $\tilde{p}_{\nu A}^r$ include the mass polarity therein. This indicates that periodical change of mass sign makes the steady-state vibration of radial motion under the constraints of eqs. (43) and (44).

Equations (43) and (44) indicate that the cylindrically radial momentums on the *x*-*y* plane are comparable to the angular ones around the *z*-axis. Since the cylindrical radii need to be positive, the radial vibration should take place frequently. This suggests that mass polarity should change once a rotation: the mass and radial-momentum polarities vary at the time of half rotation.

The square of angular momentum is defined as

$$(\lambda_{ls})^2 = \sum_k \left(\lambda_{ls}^k\right)^2 = -l_s^{sq} - l_s^{cr} \,,$$
$$l_s^{sq} = \sum_k \left(l_{Vs}^k l_{Vs}^k + l_{As}^k l_{As}^k\right), \quad l_s^{sq} = \sum_k \left(l_{Vs}^k l_{As}^k + l_{As}^k l_{Vs}^k\right).$$

The commutator is given by

$$\begin{aligned}
&(\lambda_{ls})^2 \lambda_{L-,sp} - \lambda_{L-,sp} (\lambda_{ls})^2 \\
&= 2\sum_k \sigma_k \left\{\tilde{p}_{Vs}^k + \left(\tilde{p}_{Vs}^{k1} l_{Vs}^{k2} - \tilde{p}_{Vs}^{k2} l_{Vs}^{k1}\right) + \left(\tilde{p}_{Vs}^{k1} l_{As}^{k2} - \tilde{p}_{Vs}^{k2} l_{As}^{k1}\right)\right\} \\
&\quad + 2\sum_k \sigma_k \left\{\tilde{p}_{As}^k + \left(\tilde{p}_{As}^{k1} l_{As}^{k2} - \tilde{p}_{As}^{k2} l_{As}^{k1}\right) + \left(\tilde{p}_{As}^{k1} l_{Vs}^{k2} - \tilde{p}_{As}^{k2} l_{Vs}^{k1}\right)\right\}.
\end{aligned}$$

The right-hand-side terms produce the following condition in a vector form:

$$\left(\left\{\tilde{p}_{Vs}^k\right\} + \left\{\widehat{p}_{As}^k\right\}\right) + \left(\left\{l_{Vs}^k\right\} + \left\{l_{As}^k\right\}\right) \times \left(\left\{\tilde{p}_{Vs}^k\right\} + \left\{\widehat{p}_{As}^k\right\}\right) = 0 \,. \tag{45}$$

Some algebra on this equation leads to the expression by the use of the polar coordinate for the rotational motion part:

$$\sum_\nu C_{\nu V} \begin{pmatrix} -\tilde{\partial}_{\nu V}^r - r_{\nu V}^{-1} (l_{\nu V})^2 \\ \tilde{\partial}_{\nu V}^r \tilde{\partial}_{\nu V}^\theta \\ s^{-1} \theta_{\nu V} \tilde{\partial}_{\nu V}^r \tilde{\partial}_{\nu V}^\phi \end{pmatrix} + \sum_\nu C_{\nu A} \begin{pmatrix} -\widehat{\partial}_{\nu A}^r - r_{\nu A}^{-1} (l_{\nu A})^2 \\ \widehat{\partial}_{\nu A}^r \widehat{\partial}_{\nu A}^\theta \\ s^{-1} \theta_{\nu A} \widehat{\partial}_{\nu A}^r \widehat{\partial}_{\nu A}^\phi \end{pmatrix} = 0 \,, \tag{46}$$

$$C_X = \begin{pmatrix} \sin\theta_X \cos\phi_X & \cos\theta_X \cos\phi_X & -\sin\phi_X \\ \sin\theta_X \sin\phi_X & \cos\theta_X \sin\phi_X & \cos\phi_X \\ \cos\theta_X & -\sin\theta_X & 0 \end{pmatrix}, \quad X = \nu V \text{ or } \nu A \,,$$

where the final values are expressed by vectors in the Cartesian coordinate in eq. (46). For the wave functions taken in the next section, the expected values in the *x* – and *y*-directions easily become to zero due to the average on ϕ_X in C_X. In addition, the expected values in *z*-direction go to zero by the average on the variable θ_X. Then, eq. (45) gives the constraint for the *z*-direction straight motion arising by the angular motion as

$$\tilde{p}_{Vs}^{3s} = \tilde{p}_{0V}^{3s} + \tilde{p}_{2V}^{3s} + \tilde{p}_{3V}^{3s} = 0 \,. \tag{47}$$

Therefore, the straight motion in the V-type canonical conjugate momentum of particle 1*b* coincides to that of the whole neutrino system.

7. Constituent Motion

The discussion based on eqs. (43) and (44) suggests that the radius of orbital motion should always be either increasing or decreasing. We take wave functions, which are parameterized by the use of average position $\bar{x}_{\nu X}^{k}$ and average canonical conjugate momentum $\bar{p}_{\nu X}^{k}$ for particles 1, 0, 2 and 3. It is noted that the canonical conjugate momentum $p_{1X\pi}^{\mu}$ of particle 1 (X=V or AV, $\pi = +\,\mathrm{or}\,-$) is formed by sum of particle $1a\pi$ motion $p_{1aX\pi}^{\mu}$ and its common-base particle $b\pi$ motion $p_{bX\pi}^{\mu}$ as $p_{1X\pi}^{\mu} = p_{1aX\pi}^{\mu} + p_{bX\pi}^{\mu}$. The canonical conjugate momentum $p_{\nu X}^{\mu}$ of particle $\nu = 0,2,3$ has contributions of particle $b\pi_{\nu}$ and particle νa. We, at first, consider the motion of particle $1a\pi$ and that of $\nu = 0,2,3$. The wave functions are taken as a product of spatial and time ones: the spatial wave function is made of the plane wave traveling in the *z*-direction, and the spherical wave with spherical harmonic functions, while the time wave function is given by the plane wave traveling in the *ct*-direction and the Gaussian function expressing oscillation. The linear combination of spherical harmonic functions with definite *m* stands for deviation from the reference point moving in the *z*-direction.

$$\psi_{\nu X} = \exp\left(\bar{p}_{\nu X}^{3s} x_{\nu X}^{3s}\right)\psi_{\nu X,sp}^{m} \bullet \exp\left(\bar{p}_{\nu X}^{0s} x_{\nu X}^{0s}\right)\psi_{\nu X,tm} ,$$

$$\psi_{\nu X,sp}^{m} = \sum_{l} a_{\nu Xl} R_l\left(x_{\nu X}^{r};\bar{x}_{\nu X}^{r}\right)\left(x_{\nu X}^{r}\right)^{-1}\exp\left(\bar{p}_{\nu X}^{r} x_{\nu X}^{r}\right) Y_l^{m}\left(\theta_{\nu X},\phi_{\nu X}\right), \tag{48}$$

$$R_l\left(x_{\nu X}^{r};\bar{x}_{\nu X}^{r}\right) \approx N_{\nu X,l}\sin^{l+1}\left(\frac{\pi}{2}\frac{x_{\nu X}^{r}}{\bar{x}_{\nu X}^{r}}\right),\ N_{\nu X,l} = \pi^{1/4}\sqrt{\frac{1}{2\bar{x}_{\nu X}^{r}}\frac{\Gamma(l+2)}{\Gamma(l+3/2)}},\ x_{\nu X}^{r} = 0 \sim 2\bar{x}_{\nu X}^{r},$$

$$\psi_{\nu X,tm} = \left(2\pi\left(\bar{x}_{\nu X}^{0}\right)^{2}\right)^{-1/4}\exp\left(-\frac{\left(x_{\nu X}^{0}\right)^{2}}{4\left(\bar{x}_{\nu X}^{0}\right)^{2}}\right)\exp\left(\bar{p}_{\nu X}^{0} x_{\nu X}^{0}\right),$$

where the basic straight motion in the *z*-direction is indicated by the superscript 3*s*, and $R_l\left(x_{\nu X}^{r};\bar{x}_{\nu X}^{r}\right)$ approximates the first peak of the lth-order spherical Bessel function. The radial parameter $\bar{x}_{\nu X}^{r}$ gives the approximate midpoint in the radial distribution. The wave function for particle $1aX\pi$ has the same form with the notation of $\nu X \rightarrow 1aX\pi$. For particles $\nu = 0,2,3$, we set the relation of

$$\bar{x}_{\nu X}^{r} = \sqrt{(\bar{x}_{\nu aX}^{r})^{2} + (\bar{x}_{bX\pi_{\nu}}^{r})^{2}}, \quad \bar{x}_{\nu X}^{3s} = \bar{x}_{1aX\pi_{\nu}}^{3s},$$

$$\bar{x}_{\nu X}^{0} = \sqrt{(\bar{x}_{\nu aX}^{0})^{2} + (\bar{x}_{bX\pi_{\nu}}^{0})^{2}},$$

for spatial and time motions, respectively.

Spherical harmonic functions on the canonical conjugate momentums are assigned for the V- and AV-motions in **Table 2**. At the transition, mass polarities for particles 0, 2 and 3 are supposed to change almost simultaneously. We assign the system to two types of mass allocation, i.e. cases 1 and 2. Particles 0 and 3 exist in the positive state in case 1, while they reside in the negative one in case 2. Radii of V-type orbital motions are in the expanding state and those of AV-type also expanding in case 1, while all the radius behaviors are reversed in case 2. The abbreviation in linear combination is shown below the table.

Table 2 Allocation of angular momentums in U^c. The spin part l_s is shared by orbital motions of particles 0, 2 and 3. Symbols for linear combination is listed below.

Case 1

V	l_{1V}	l_{1aV}	l_{0V}	l_{2V}	l_{3V}	l_s
Pos.	Y_2^{-2}	$Y_{3/2\&5/2}^{-3/2}$	$Y_{1\&2}^{-1}$		$Y_{1\&2}^{0}$	Y_s^0
Neg.	Y_2^{1}	$Y_{1/2\&3/2\&5/2}^{1/2}$		$Y_{1\&2}^{1}$		
AV	l_{1A}	l_{1aA}	l_{0A}	l_{2A}	l_{3A}	l_s
Pos.	Y_2^{-1}	$Y_{1/2\&3/2\&5/2}^{-1/2}$	$Y_{1\&2}^{-1}$		$Y_{1\&2}^{0}$	
Neg.	Y_2^{2}	$Y_{3/2\&5/2}^{3/2}$		$Y_{1\&2}^{1}$		$Y_s^{-1/2}$

Case 2

V	l_{1V}	l_{1aV}	l_{0V}	l_{2V}	l_{3V}	l_s
Pos.	Y_2^{-1}	$Y_{1/2\&3/2\&5/2}^{-1/2}$		$Y_{1\&2}^{-1}$		Y_s^0
Neg.	Y_2^{2}	$Y_{3/2\&5/2}^{3/2}$	$Y_{1\&2}^{1}$		$Y_{1\&2}^{0}$	
AV	l_{1A}	l_{1aA}	l_{0A}	l_{2A}	l_{3A}	l_s
Pos.	Y_2^{-2}	$Y_{3/2\&5/2}^{-3/2}$		$Y_{1\&2}^{-1}$		
Neg.	Y_2^{1}	$Y_{1/2\&3/2\&5/2}^{1/2}$	$Y_{1\&2}^{1}$		$Y_{1\&2}^{0}$	$Y_s^{-1/2}$

$$\text{Symbol in the table}:\ Y_{1\&2}^{m} : a_{\nu X1} R_1 Y_1^m + a_{\nu X2} R_2 Y_2^m$$
$$Y_s^0 : a_{s1/2} R_{1/2} Y_{1/2}^{1/2} + a_{s3/2} R_{3/2} Y_{3/2}^{-1/2}, \quad Y_s^{-1/2} = Y_{1/2}^{-1/2}$$

The wave function for each particle of $\nu = 0, 2, 3$ has orbits of V- and AV-type in the linear combination form. The wave function $\psi_{\nu V}$ for the V-orbit is taken to be orthogonal to $\psi_{\nu A}$ for AV-one. This is accomplished by the phase difference of $\pm\pi/2$ between coefficient vectors $(a_{\nu V1}, a_{\nu V2})$ and $(a_{\nu A1}, a_{\nu A2})$. In this situation, the combination of particles of $\nu = 0, 2, 3$ and the spin part produces an effective orbital state Y_2^0 and spin -1/2, producing a square of combined angle momentums of 6.5. In contrast, the particle 1 has the canonical conjugate orbits $Y_2^{\pm 2}$ and $Y_2^{\pm 1}$ in the table. These orbits of particle 1 and the effective orbital state Y_2^0 of particles 0, 2 and 3 should constitute a closed shell of Y_0^0. Therefore, the neutrino system finally has the spin of -1/2.

Since $l_{Vs}^3 + l_{As}^3 = -(1/2)i$, the radii of particles 0, 2, 3 are increasing in case 1, according to eqs. (43) and (44). At the half rotation, the masses changes to different polarities and makes case 2. The angular motion continues with radii decreasing, to reach the original positions. Such a motion is considered to be a kind of steady-state movement, in spite of radius variation. To achieve the steady-state motion, it is natural to impose a constraint on average radius in eq. (48), so that the phases in de Broglie waves at the half rotation may have a relation of either the same or integer-fold to a reference motion. The constraint is required for producing a certain circulation period for particles 0, 2 and 3.

Mass terms (22) and (24) are converted into Hamiltonian with potential terms eq. (27) and (33) with regards to the above average quantities:

$$H_{pt} = H_{b,pt} + H_{\nu,pt} + \sum_j \lambda_j (\text{constraint - j}),$$

The electromagnetic energies at particle positions by eq. (35) serve as kinetic mass energies. The Hamiltonian for cases 1 and 2 produces the steady-state solution and the neutrino mass which is consistent to the two cases. A test calculation was made for a neutrino of total energy 1MeV. A rough estimate of constituent mass is listed in **Table 3**, where the average radius of positive motion of particle 1*b* was commonly set at a typical value of 10^5 fm in both cases 1 and 2. In addition, the mass of particle 0 was assumed to be the same as that of particle 1 in case 1, whereas the mass of particle 1 was postulated to be equal to that of particle 2 in case 2. Constituent mass values were searched by a Monte Carlo method with consideration of preceding constraints, and the total mass was chosen to be close to the week-electromagnetic self-energy of the neutrino system. This rough estimate gives a total mass of eV level. General calculation with possible less number of approximations will lead to a more realistic neutrino mass.

Table 3 Calculation example of constituent mass in units of keV for neutrino of total energy 1 MeV.

	mass#1	mass#0	mass#2	mass#3	total mass
Case1	1.728×10^{1}	1.728×10^{1}	-3.457×10^{1}	3.032×10^{-2}	2.845×10^{-2}
Case2	7.666	-1.524×10^{1}	7.666	-3.66×10^{-3}	9.275×10^{-2}

8. Discussion

The auxiliary field $\hat{B}_A^0$ influences to formation of the constituent masses both in the electromagnetic self energy and the kinetic mass-energy. There may be some natural materials to be capable of generating $\hat{B}_A^0$. When neutrinos are incident to such material region, the mass formation mechanism of neutrinos may be disturbed due to contradiction of the electromagnetic energy with the kinetic mass-energy. In that case, a neutrino is expected to be dissociated into two groups: it is separated into particle 1 and a group of particles 0, 2 and 3 because of disappearance of $Q - Q_d$ interaction. In fact, such reaction was supposed to be found [8)] in our group. Biological product of raw silk seems to dissociate environmental neutrinos through the generation of $\hat{B}_A^0$, to make a appreciable voltage generation by the weak-charge interaction.

9. Summary

Neutrino structure was studied on the basis of weak-charge and weak-electric-moment interaction. The constituent particles were assumed to basically reside in individual subspaces through the transformation by gamma matrices. The extended Dirac equation consists of linear operators based on $\gamma^0\gamma^\mu$ and $\gamma^0\gamma^5\gamma^\mu$ matrices, different from γ^μ and $\gamma^5\gamma^\mu$ ones. The exchange properties between the linear operators and angular momentum ones give the view that the motion of spin -1/2 system should always include vibration of radii and constituent-particle mass-polarity change. The property of gamma matrices imposes some constraints on the constituent particle motion: Particularly the momentums in specified directions need to be canceled between V- and AV–types. The cancellation in specified directions accompanies the complex-type-unfit motions. The system having momentums of $\gamma^0\gamma^\mu$ and $\gamma^0\gamma^5\gamma^\mu$ matrices offer a special flexible feature in the potential generation through the unit matrix in the time direction. The complex-type-unfit motions generate the auxiliary field under the Fermi gauge, and the auxiliary field works to make the electromagnetic self energy. The electromagnetic self energy serves as kinetic mass for the constituent particle motion. It was considered that such weak-charge and weak-electric-moment interaction system readily disintegrated by the external auxiliary field.

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