Memoirs of the Faculty of Engineering, Kyushu University, Vol. 68, No.2, June 2008

# Development and Verification of a Three Dimensional Density Dependent Solute Transport Model for Seawater Intrusion

by

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(Received May 9, 2008)

**Abstract**

Seawater intrusion is identified as a critical issue that needs to be addressed in coastal hydrology, due its irreversible effect on the water quality of coastal freshwater aquifers. Due to the density differences in seawater and freshwater, the density dependent flow phenomenon is dominant seawater intrusion. Three dimensional numerical model is necessary to address the complex hydro-geological formations. Therefore, it is important to develop a three dimensional simulation model to understand the density dependent solute transport process taking place in the seawater intrusion problem. The model developed in this study is based on the transition zone approach which couples the groundwater flow and mass transport equation to solve the density dependent flow. The finite difference method is used as the numerical method to solve the partial difference equations of groundwater flow and solute transport. The method of characteristic is applied to solve the advection term of mass transport equation. Therefore, in this paper, a valued attempt has been taken to simulate the seawater intrusion in the three dimensional point of view under the density dependent flow effect. Motooka area of Fukuoka Prefecture Japan which is a coastal area is selected as an application area for the developed model to emphasize seawater intrusion and justify the model. This area is affected by seawater intrusion and so far numerical study of seawater intrusion under the density dependent flow concept has not performed.

**Keywords:** Density dependent flow, Method of characteristic, Seawater intrusion, Up-coning

## 1. Introduction

In various natural and engineered systems, the density dependent flow process is identified as a dominant factor which governs the fate of flow. One of the typical applications of density dependent flows is seawater intrusion in exploited coastal aquifers. In hydrogeology, seawater intrusion and up-coning processes are subjects of specific concern [1)]. Seawater intrusion can simply be defined as the inflow of seawater into a coastal freshwater aquifer. The major reason for this phenomenon is the abstraction of groundwater in exceeded limits of natural equilibrium between freshwater and seawater, that have been merged together in a coastal aquifer. Due to the over exploitation of groundwater the seawater starts to encroach into the aquifer to keep the potential balance.

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Seawater intrusion is an important coastal hydrological problem to address since 70% of the earth's population lives near coastal areas, and 95% of the earth's water lies in the oceans and seas at high levels of salinity. The study of seawater intrusion has been quantitatively researched by numerous authors over the world [2), 3), 4), 5), 6), 7)]. Due to its practical importance, the numerical modeling of seawater intrusion has attracted remarkable attention in the water resource literature over the last 20 years [1)]. Two general approaches, i.e. the sharp interface approach and the transition zone approach, have been used to analyze seawater intrusion in coastal aquifers. Coupled freshwater and seawater flow equations are solved simultaneously when a sharp interface is involved in a coastal zone. Models that concern density dependent flow or transient zone in a coastal region require simultaneous solution of governing water flow and solute transport equations[8)]. When an assessment of the salt concentration in both local and regional flow systems is desired, the transition zone density dependent flow approach is required. To study seawater intrusion, density dependent groundwater flow dynamics are needed to simulate the flow in the transition zone between freshwater and seawater[2)]. The first attempts to model the density-dependent miscible seawater/fresh water ware carried out by Henry[9)] and Pinder and Cooper[10)].

Mathematical modeling of seawater intrusion plays a key role in the understanding, monitoring, remediation and development of any coastal aquifer where groundwater is being exploited. Most importantly mathematical models should have the ability to simulate the three-dimensional seawater intrusion since most of the coastal aquifers are complex in nature[11)]. The development of any numerical model should be capable of simulating the phenomenon correctly and of cause the model has to have the capability to update to use it as a management tool to optimize the exploitation of a coastal aquifer.

This paper describes the three dimensional density dependent solute transport numerical model which simulates the impact of groundwater pumping in a coastal aquifer, inspired by the condition found in the Motooka coastal area of Fukuoka prefecture Japan. Since Motooka is a groundwater based agricultural area the salinity levels of the pumped groundwater is very important. This study addresses the seawater intrusion issue of Motooka under the numerical simulation model to illustrate the phenomena. A groundwater recharge model developed by Tsutsumi et al[12)] 2004 was used as a regional model to generate recharge rate and water tables which are used as initial and boundary conditions for the density dependent three dimensional model. Comparison of measured and calculated values of water table levels, electric conductivity variation with the depth at electric conductivity observation wells, electric conductivity variation of pumped groundwater with pumping rates, up-coning and movement of transition zone are illustrated.

## 2. Description of the Site

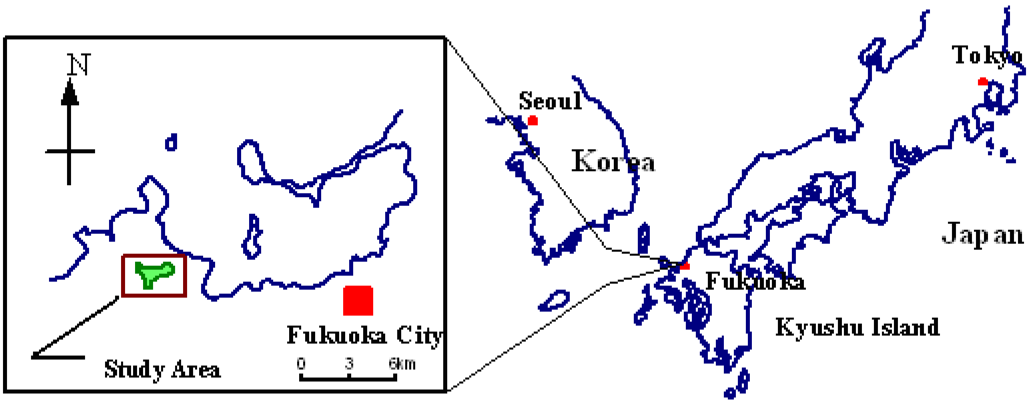
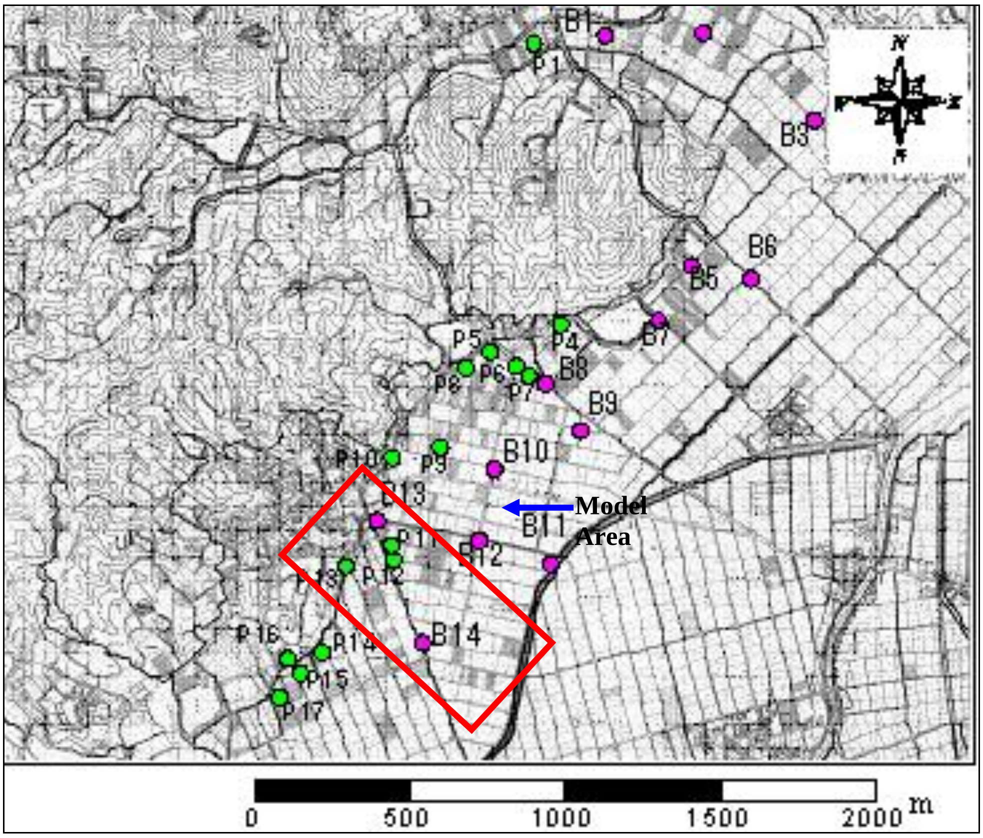



**Fig. 1** Location of Fukuoka and study area.

The developed model was applied to simulate the seawater intrusion in Motoka area; which is located in the western region of Fukuoka of Kyushu Island of Japan as shown in **Fig. 1**. Groundwater is the main source of water for drinking, winery and greenhouse cultivation in this area. The cumulative amount of use is 1,100 ~ 1,200 $m^3$/day for agriculture, household and winery in Motooka region. **Figure 2** shows the groundwater pumping wells and electric conductivity monitoring wells distribution in Motooka. More than 20 wells are functioning for groundwater pumping and the wells close to the sea have shown an increase of salinity.

In the Motooka area, lowland alluvial plain area is being used for paddy cultivation and greenhouse farming whilst the mountain side is allocated as a residential area. Mountain side contributes as the recharge area for the freshwater aquifer. The elevation of the ground surface ranges from 0.3 m at the lowest point to about 100 m a.m.s.l at the highest point. Under this plain, a shallow unconfined aquifer has developed and it is partially affected by seawater intrusion. The thickness of the unconfined aquifer under the low land is approximately 50 m [12].




- Pumping wells, - Electric conductivity monitoring wells

**Fig. 2** Groundwater pumping wells and electric conductivity monitoring wells distribution.

## 3. Model Development

### 3.1 Mathematical model

This section describes mathematical equations which govern the density dependent groundwater flow and transport of solute in a porous media. The basic equations used to develop the mathematical model are described below.

Darcy's velocity;

$$u = -k\ (h) \cdot \frac{\partial h}{\partial x} \tag{1}$$

$$v = -k\ (h) \cdot \left( \frac{\partial h}{\partial y} + \frac{\rho}{\rho_f} \right) \tag{2}$$

$$w = -k\ (h) \cdot \frac{\partial h}{\partial z} \tag{3}$$

where, $u$, $v$ and $w$ are the Darcy's velocities in $x$, $y$, and $z$ directions respectively. $k(h)$ is the hydraulic conductivity. $h$ is pressure head. $\rho$ and $\rho_f$ are the water density and freshwater density respectively. In this model $y$ direction is taken as the vertical axis in the orthogonal $x$, $y$ and $z$ coordinate system.

Continuity equation for groundwater flow;

$$(C_W + \alpha S_s) \cdot \frac{\partial h}{\partial t} = -\left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \tag{4}$$

where, $S_s$ is the specific storage coefficient, $C_w$ is the specific moisture capacity, $\alpha$ is a dummy which takes 0 in unsaturated condition and 1 in the saturated condition.

Mass transport equation for advection and dispersion;

$$\begin{aligned} &\frac{\partial(\theta C)}{\partial t} + \frac{\partial(u'\theta C)}{\partial x} + \frac{\partial(v'\theta C)}{\partial y} + \frac{\partial(w'\theta C)}{\partial z} = \frac{\partial}{\partial x}(\theta D_{xx}\frac{\partial C}{\partial x} + \theta D_{xy}\frac{\partial C}{\partial y} + \theta D_{xz}\frac{\partial C}{\partial z}) + \\ &\frac{\partial}{\partial y}(\theta D_{yx}\frac{\partial C}{\partial x} + \theta D_{yy}\frac{\partial C}{\partial y} + \theta D_{yz}\frac{\partial C}{\partial z}) + \frac{\partial}{\partial z}(\theta D_{zx}\frac{\partial C}{\partial x} + \theta D_{zy}\frac{\partial C}{\partial y} + \theta D_{zz}\frac{\partial C}{\partial z}) \end{aligned} \tag{5}$$

where, $\theta$ is the volumetric moisture content. $D_{xx}$, $D_{yy}$, $D_{zz}$, $D_{xy}$, $D_{xz}$, $D_{yx}$, $D_{yz}$, $D_{zx}$ and $D_{zy}$ are dispersion coefficients, which are dependent on the real pore velocity, are represented by equation (6).

$$\left.\begin{aligned} \theta D_{xx} &= \frac{\alpha_L u'^2}{V} + \frac{\alpha_T w'^2}{V} + \frac{\alpha_T v'^2}{V} + \theta D_M \\ \theta D_{yy} &= \frac{\alpha_T u'^2}{V} + \frac{\alpha_T w'^2}{V} + \frac{\alpha_L v'^2}{V} + \theta D_M \\ \theta D_{zz} &= \frac{\alpha_T u'^2}{V} + \frac{\alpha_L w'^2}{V} + \frac{\alpha_T v'^2}{V} + \theta D_M \\ \theta D_{xy} &= \theta D_{yx} = \frac{(\alpha_L - \alpha_T) u' v'}{V} \\ \theta D_{xz} &= \theta D_{zx} = \frac{(\alpha_L - \alpha_T) u' w'}{V} \\ \theta D_{yz} &= \theta D_{zy} = \frac{(\alpha_L - \alpha_T) w' v'}{V} \end{aligned}\right\} \tag{6}$$

where, $u'$, $v'$ and $w'$ are the components of real pore velocities in $x$, $y$ and $z$ directions calculated by $u'=u/\theta$, $v'=v/\theta$ and $w'=w/\theta$. $\alpha_L$ and $\alpha_T$ are the microscopic dispersion legths for longitudinal and transverse directions, $D_M$ is the fluid molecular diffusion coefficient, and $|V| = \sqrt{u^2 + w^2 + v^2}$ is the magnitude of the velocity vector.

Equations (4) and (5) are coupled directly via a linear equation of state, in which the density of groundwater was assumed to be directly proportional to the salinity, while other factors influencing the density were neglected.

$$C = \{ (\rho - \rho_f)/(\rho_s - \rho_f) \} \times 100.0 \tag{7}$$

where, $C$ corresponds to the concentration of seawater. $C$ = 100% represents the seawater while $C$ = 0% represents the fresh water. $\rho_s$ is the density of seawater.

### 3.2 Conceptual model

Bear and Veruijt[13] stated that the conceptual model consists of a set of assumptions that reduce the real problem and real domain to a simplified version that is acceptable in view of the objectives of the modeling. Bredehoeft[14] suggested that the conceptual model is the basic idea, or construct, of how the system or processes operates; it forms the basic idea for the model (or theory). Technically, a conceptual model is nothing more than a set of assumptions that relate to boundary conditions, initial conditions, geological description, flow regime, source/sink distribution, hydraulic conductivity distribution, transport parameters and other flow and transport parameters [15]. **Table 1** demonstrates the boundary conditions of pressure heads and concentration applied for the model. **Figure 3** shows the six boundaries of the model. Time dependent pressure head boundary conditions were applied to the ABCD, EFGH and ADEF boundaries as shown in **Fig. 3** by coupling the results obtained from quasi – three dimensional groundwater flow model developed by Tsutsumi et al[12]. The bottom boundary was considered as an impermeable zero flux boundary. For the ground surface timely variable recharge $R(x, z, t)$ boundary condition was applied. For the seaside boundary (BCHG) constant pressure head boundary condition was applied while the concentration boundary of the seaside was kept as a velocity direction dependent as shown in the **Table 1**. **Figure 4** illustrates the topographical features, pumping wells (P1, T8 and T9), electric conductivity observation well (B14), water table observation well (WL8) and the model boundaries.

**Table 1** Boundary conditions of the numerical model.

| Boundary | Pressure Head | Concentration |
|---|---|---|
| ABCD | $h_{ABCD}(t) = H_{ABCD}(t) - y$ | $\partial C / \partial y = 0.0$ |
| EFGH | $h_{EFGH}(t) = H_{EFGH}(t) - y$ | $\partial C / \partial y = 0.0$ |
| ADEF | $h_{BCGF}(t) = H_{BCGF}(t) - y$ | $C = 0.0$ % |
| BCHG | $h_p = (H_s - y) \cdot \dfrac{\rho}{\rho_f}$ | $u > 0, \dfrac{\partial C}{\partial x} = 0$<br>$u < 0, C = 100\%$ |
| ABGF | $-k(h)\left[\dfrac{\partial h}{\partial y} + \dfrac{\rho}{\rho_f}\right] = 0$ | $\partial C / \partial y = 0.0$ |
| CDEH | $-k(h)\left[\dfrac{\partial h}{\partial y} + \dfrac{\rho}{\rho_f}\right] = -R(x,z,t)$ | $\partial C / \partial y = 0.0$ |

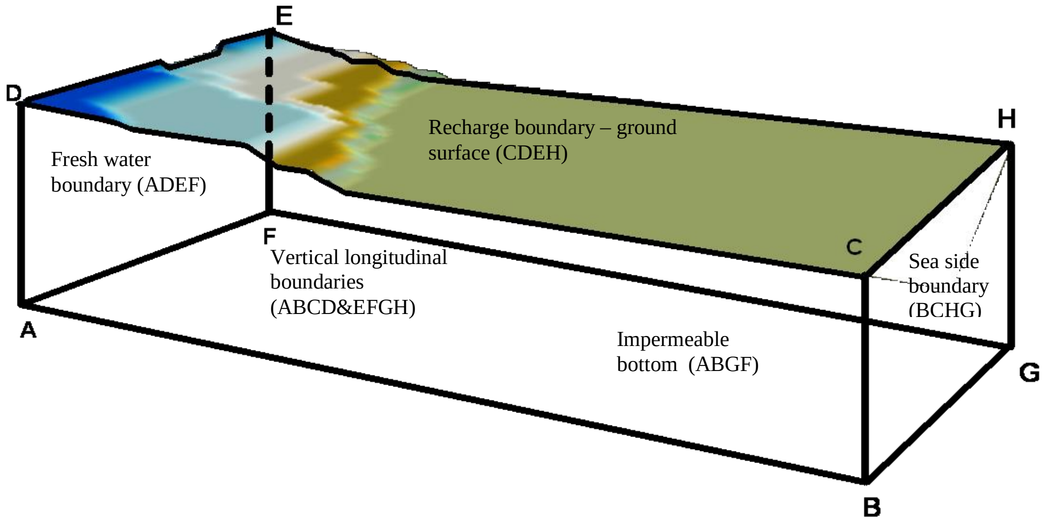



**Fig. 3** Boundaries of the model.

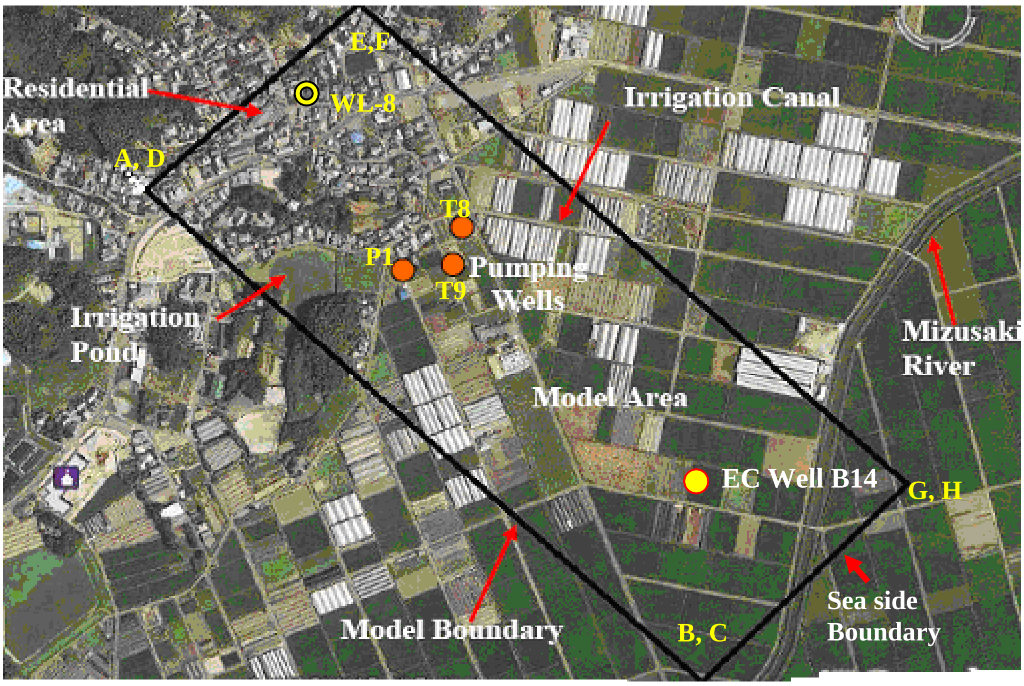



**Fig. 4** Topographical map of model area.

### 3.3 Numerical simulation

The three dimensional density dependent solute transport model, which is explained in this paper, uses the finite difference method to solve the partial difference equations of flow and solute transport. Chemical, biological decay and adsorption are not concerned in this model. The salient feature of the model can be summarized as follows. The transient groundwater flow equation (4) is solved by an implicit finite difference method using an iterative successive over relaxation (SOR) technique. The solute transport equation (5) is solved in two step processes. Whereas the advection term is computed by the method of characteristic (MOC), the dispersion term is calculated by an explicit finite difference method. The concentration of each grid point is evaluated by associating mass to a number of tracer particles within a cell which are moved within the Lagrangian frame along the flow characteristics and subsequently interpolated to the grid points of the cells. In this simulation the maximum number of particles assigned for each cell is 8. This number was selected after compromising the preciseness and calculation time. After locating 8 particles for each grid, points, the velocity and concentration information of the grid point are assigned to those particles which belong to the particular grid. Then the particles are allowed to move with the flow for one time step. After one time step the average concentration of moved particles which are in the particular grid domain is assigned to the grid point. The dispersion term is then computed at these grid points. The same procedure is followed for all the grid points inside the model domain at each time step.

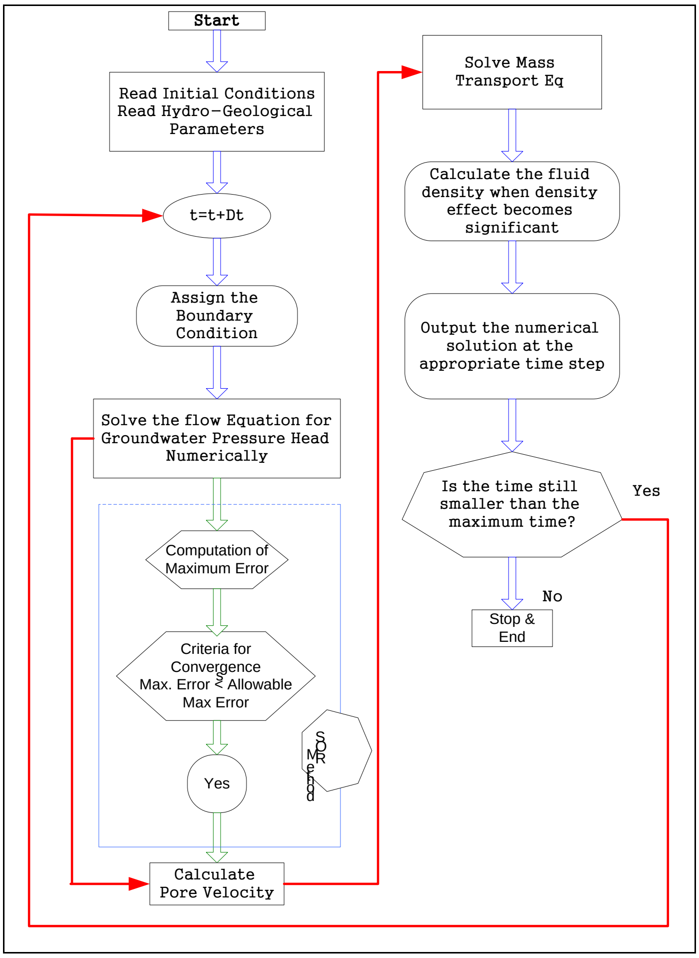



**Fig. 5** Flow chart of the numerical model.

**Figure 5** is the flow chat which describes the step by step calculation procedure adopted in this numerical model.

Since the pumping well area is considered as the more sensitive in comparison to other places of the model domain, small grid sizes are used for that area. The length of the selected area is 1056.0 m and its width is 417.0 m. The height of the model is 56.0 m. The model domain is divided into non uniform discretized grid system for *x* and *z* directions. The smallest grid sizes in the *x* and *z* directions are 4.0 m and 5.0 m respectively. For *y* direction constant grid size of 2.0 m is used. **Figure 6** demonstrates the grid distribution of the model domain.

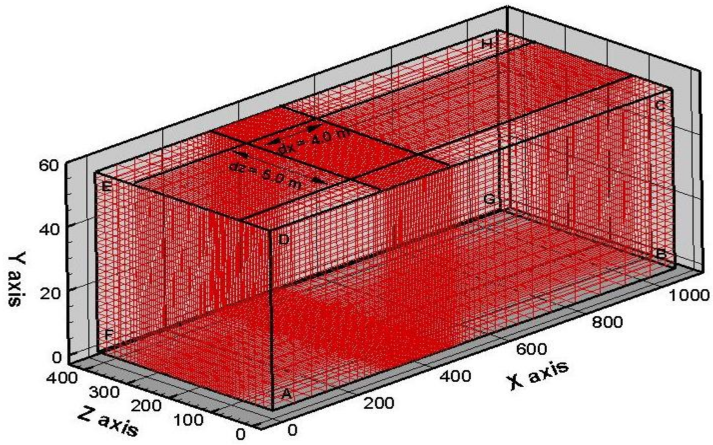



**Fig. 6** Grid distribution.

The hydro-geological parameters used in the model were obtained from borehole information, field measurements and literature [16]. The longitudinal and transverse dispersion lengths were set to 3.6m and 0.36m, respectively, while molecular diffusion was 1.0 x $10^{-9}$ $m^2$/s. The standard seawater density value of 1025.0 kg/$m^3$ was used. The density of fresh water was set to 1000.0 kg/$m^3$. 1.6 x $10^{-8}$ m/s, 6.6 x$10^{-7}$ m/s, and 4.6 x$10^{-6}$ m/s were the hydraulic conductivities of bed rock, confining layer and flow dominant region respectively. **Figure 7** demonstrates the major geological regions of the model. The time increment was set to 4 hours.

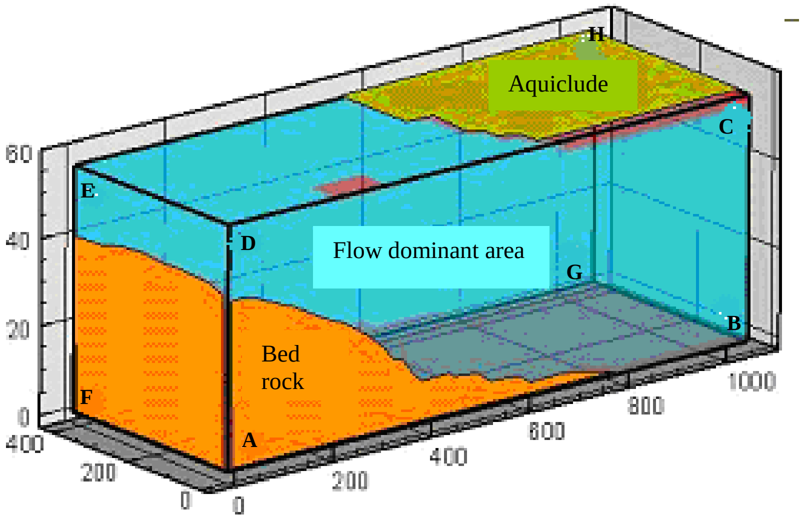



**Fig. 7** Major geological regions of the model.

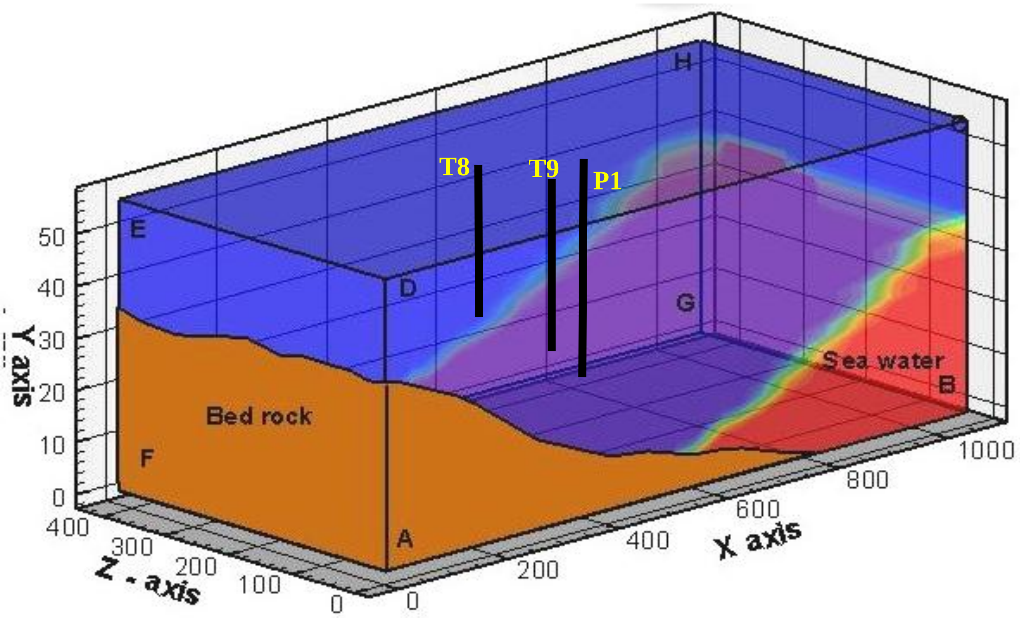



**Fig. 8** Initial conditions of the model.

## 4. Results and Discussion

**Figure 8** shows the initial distribution of concentration obtained under the natural condition without pumping. There are 3 pumping wells in the model domain (P1, T8 and T9). The boring depths of P1, T8 and T9 wells are 40 m, 20 m and 30 m, respectively. The well diameters for P1, T8 and T9 are 100 mm, 100 mm and 120 mm, respectively. Wells P1, T8 and T9 are located at 703 m, 679 m and 647 m from the sea boundary (BCHG as shown in **Fig. 3**).

In this paper attention was paid to examine the capability of the developed model to simulate the hydrological processes such as water table fluctuations, mixing zone movement and up-coning of a coastal aquifer where groundwater is exploited. The model was run for 6 years from 2002 to 2007. The obtained results were demonstrated below.

In order to verify the accuracy of the model quantitatively, the observed and calculated values of water table (2002-2007) for the water table observation well WL8 (**Fig. 4**) and electric conductivity variations with depth (2004, 2005 and 2006) at well B14 (**Fig. 4**) located inside the model domain are illustrated below.

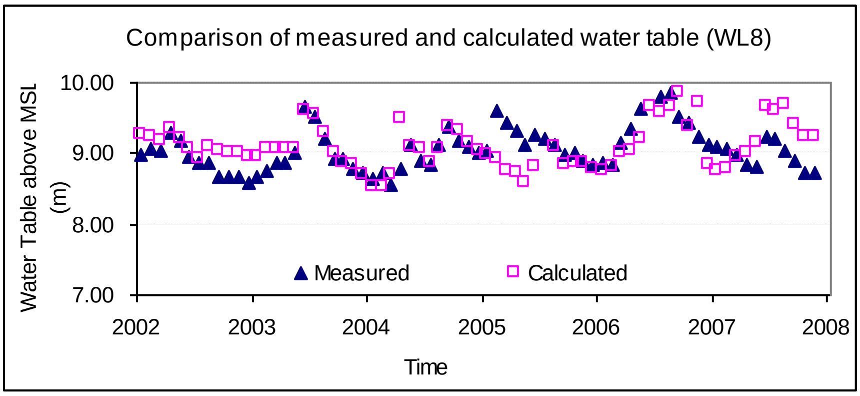



**Fig. 9** Comparison of measured and calculated water tables at observation well WL8.

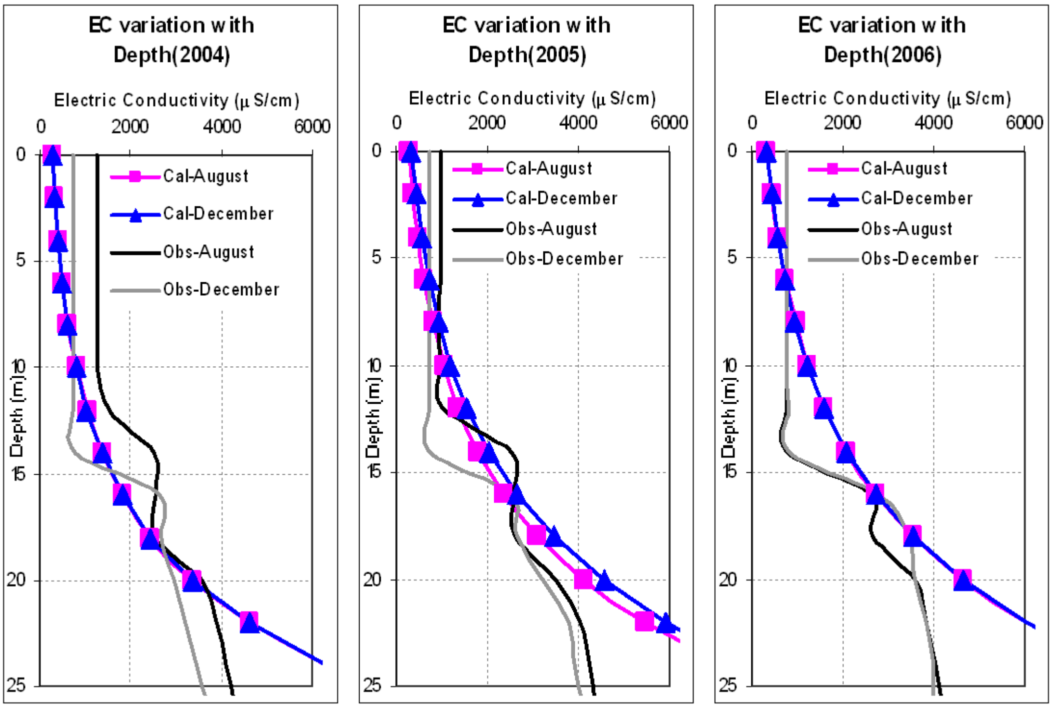



**Fig. 10** Comparison of measured and calculated electric conductivities with depth at EC well B14.

**Figure 9**, the comparison of measured and calculated water tables of well WL8 (according to **Fig. 4**) is for the time period from 2002 to 2007. The measured water tables and calculated water tables show a good agreement. Therefore the model can be used as a tool for the prediction of water tables for future water management activities. **Figure 10** shows the measured and calculated electric conductivities of electric conductivity observation well B14 for the years 2004, 2005 and 2006. Up to 20m the simulated results and measured values of electric conductivities show a reasonable agreement. But after that a deviation can be observed in the simulated results. The reason for this variation can be explained as due to the seaside boundary condition applied for this numerical simulation. The actual seaside boundary is located far from the model seaside boundary. Because of that the depth where seawater exists in the field is deeper than the seawater depth of the model. The Due to this reason numerical results of the EC well B14 shows an increased of electric conductivity values after the depth of 20 m.

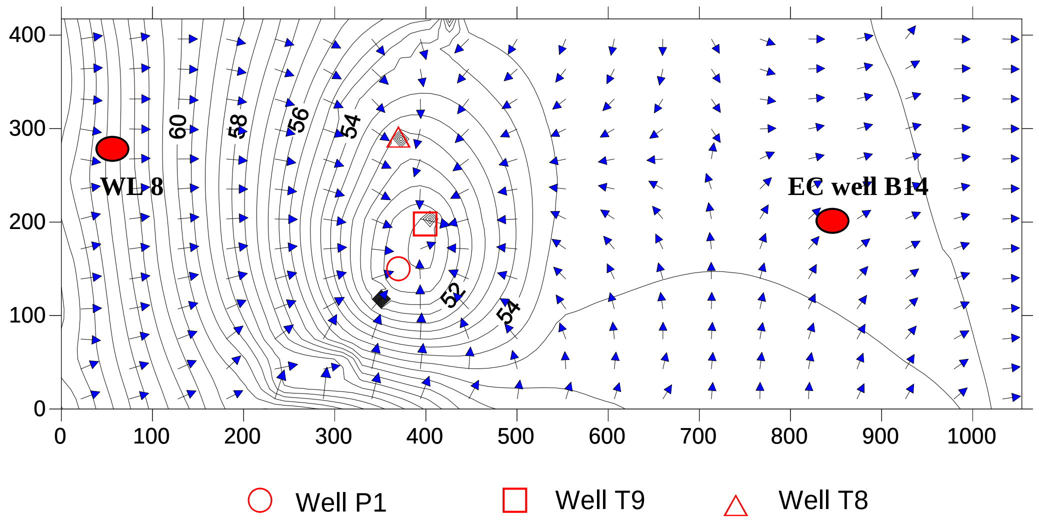



**Fig. 11** Simulated water table elevation contours and groundwater flow.

In **Fig. 11** the arrows demonstrate the groundwater flow direction at water table elevation while the lines show the contours of the water table elevations which are generated by the numerical model. The numerical result convinces that the model can simulate the groundwater flow reasonably.

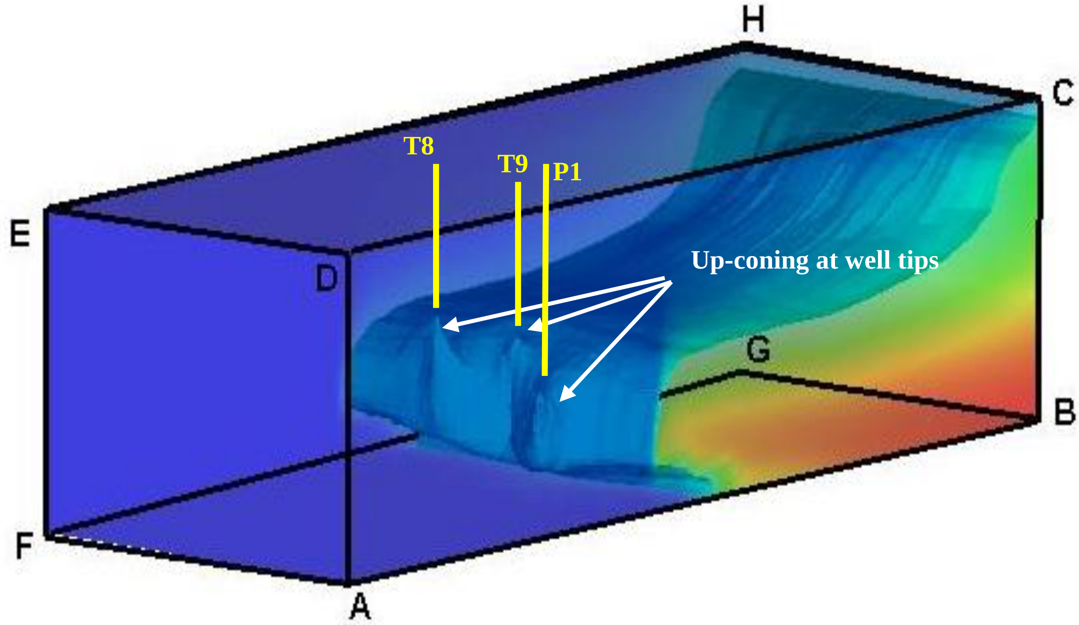



**Fig. 12** Up-coning at pumping wells.

**Figure 12** shows the up-coning phenomenon which occurred at the pumping wells simulated by the developed three dimensional model. The up-coning surface shown in **Fig. 12** was the iso-surface drawn for 5% salt concentration. It can be seen the up-coning is taking place at the tip of each well. In the Motooka area if the pumped groundwater contains high levels of salt, it will directly affect the agricultural products such as vegetables and fruits which are cultivated inside the greenhouses. According to **Fig. 12, Fig. 13, Fig. 14** and **Fig. 15** it can be clearly seen that the model is capable in simulating the up-coning satisfactorily. This model can be run for field pumping rates and get results which will be useful for farmers of the Motooka area and for the scientists who are interested in seawater intrusion in the Motooka area. **Figures 13**, **14**, and **15** explain the temporal movement of mixing zone towards the pumping well T9 from its initial position.

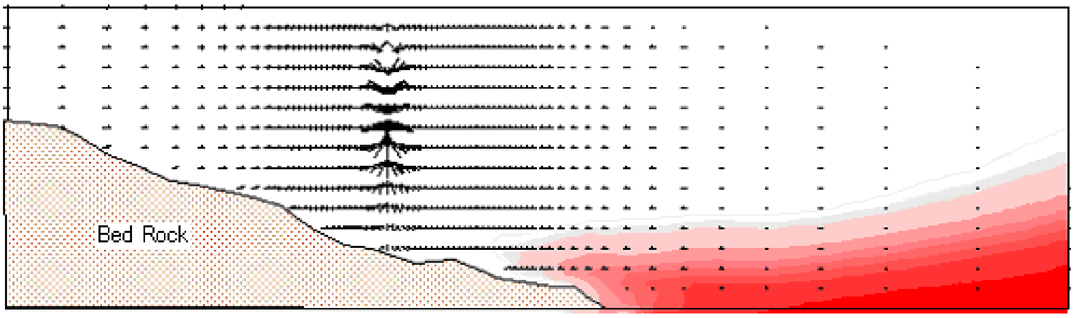



**Fig. 13** Mixing zone location after three months calculation.

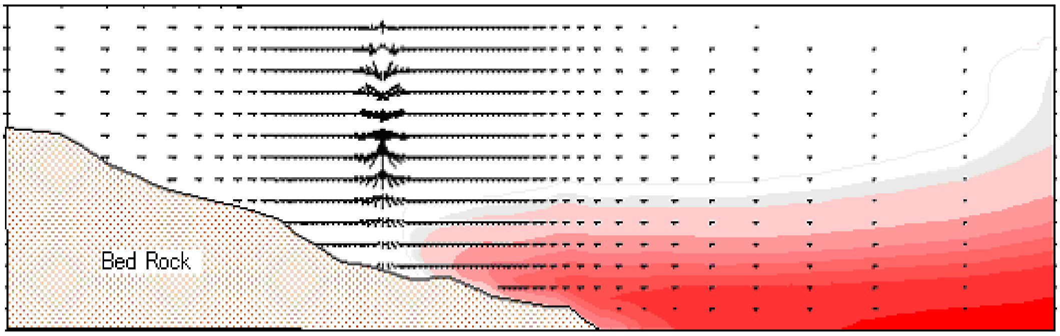



**Fig. 14** Mixing zone location after six months calculation.

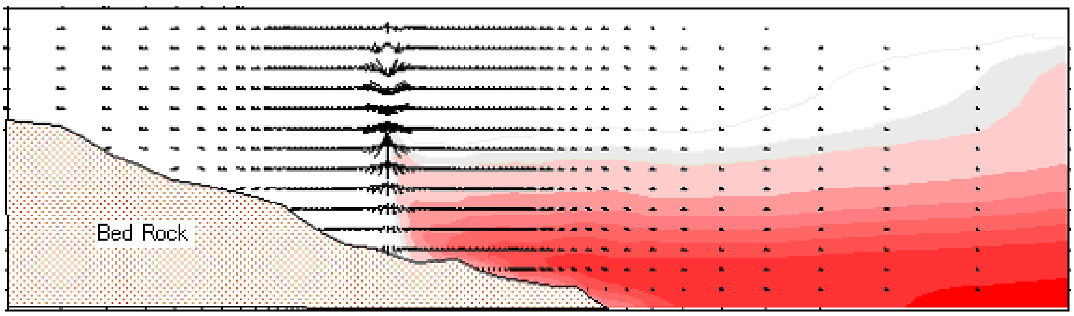



**Fig. 15** Mixing zone location after nine months calculation.

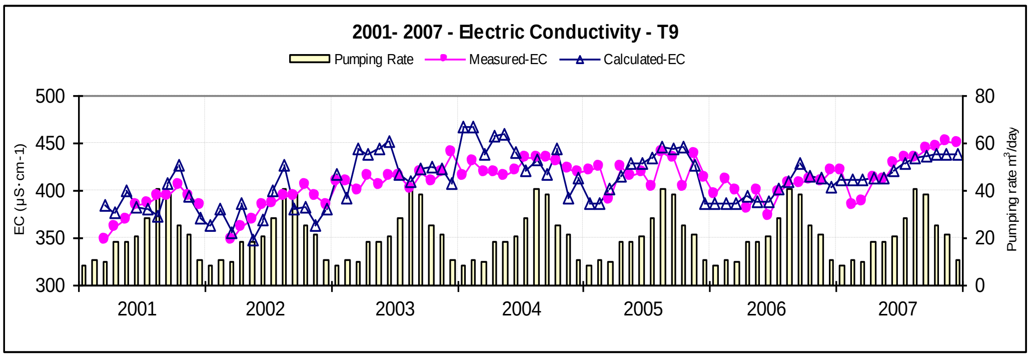



**Fig. 16** Comparison of measured and calculated electric conductivity values of pumped groundwater of well T9.

In the Motooka area, the farmers who use pumped groundwater for their agricultural purposes complain about the salinity fluctuations of the pumped groundwater. **Figure 16** demonstrates the application of the model to simulate the electric conductivity fluctuations with the field pumping rates for the well T9. The electric conductivity fluctuations are caused by the variable pumping rates. The simulation was carried out from the year 2001 to 2007. The measured and calculated electric conductivities show a reasonable agreement. It can be seen that the increment of electric conductivity values from 2001 to 2007 in measured and calculated values. It gives a clue of the threat of seawater intrusion.

## 5. Conclusion

Three dimensional numerical models can be considered as more realistic and practical than the two dimensional models due to the geological and hydrological complexities of coastal aquifers. Moreover density dependent transport models are indispensable in modeling seawater intrusion since the flow of two different densities of water is involved. In this sense, the authors found the importance of developing a three dimensional density dependent numerical model. Therefore, the objective of this paper was to discuss the development and the justification of a three dimensional density dependent solute transport model to simulate the seawater intrusion which is considered as a crucial problem in coastal aquifers where groundwater is being exploited. The transition zone induced by the dispersion process between the fresh and seawater is considered in this model. An irregular grid system is adopted in developing the three dimensional finite difference numerical model. In the course of the numerical simulation of the seawater intrusion in the Motooka area, various hydro-geological parameters and different boundary conditions are assigned. Simulations done by the model for the hydrological processes of up-coning, transition zone movement and groundwater flow were reasonably successful. Although the model adopted in this study is based on limited actual observations and typical parameter values, the numerical model of this site proved useful in demonstrating the mechanism of seawater intrusion in Motooka. More importantly, the model can be used to predict the trends of up-coning and physical movement of mixing zone in Motooka. The concepts and motivations provided by this simulation work can be used by local water management authorities to understand the seawater intrusion phenomenon that takes place in Motooka.

## Acknowledgement

The authors are grateful to the Obayashi Foundation of Japan for their generosity to provide research funds to carry out this research successfully.

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