Memoirs of the Faculty of Engineering, Kyushu University, Vol.73, No.3, December 2013

Comparison of Optimal Basis Function for the Underground Microseismic Wave Processing in Wavelet Packet Transform

by

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(Received October 28, 2013)

Abstract

To improve the accuracy of signal analysis and processing for the underground microseismic waves, an optimal basis function is indispensable in the wavelet packet transform (WPT). Based upon the microseismic wave groups monitored in a deep coal mine, wavelet bases in the Daubechies, Symlets and Coiflets families were screened, and the optimal wavelet packet basis was strictly determined by its reconstruction capability on the original wave and its conservation capability on its characteristic components. Signal reconstruction and conservation capabilities were evaluated by two parameters, root mean square error and correlation coefficient. The energy reserving capability of the optimal basis function was finally discussed to verify its superiority. The results turn out that the wavelet bases db1, sym4, sym5 and sym8 are more appropriate for the microseismic wave compared to others as their better signal reconstruction capability. Among them, basis sym5 is the optimal wavelet basis function for the microseismic wave. When processed by the wavelet basis sym5, the maximum energy components of waves are effectively reserved and the reconstructed characteristic components have the highest relevancy with that of the original wave.

Keywords: Underground coal mining, Microseismic wave processing, WPT, Optimal basis function, Energy reserving capability

1. Introduction

With the increase of the underground mining depth, more and more serious mining tremors occur and threaten the safe production. Except for the rock burst, microseism is also one kind of

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these serious mining tremors. The researches on microseism play an important role in the prevention of many mining disasters [1,2], Recently, detail analysis to the microseismic wave is becoming one of the main research interests in the prevention of rock burst hazard. Especially, the spectrum analysis related to its time, frequency and energy is an important approach for mining, describing and responding the potential characteristics of waves [3,4]. After the microseismic event, a large quantity of elastic energy releases and propagates in deep country rock in the form of body wave [5]. Microseismic wave has some specialties such as the short duration, the sudden saltation, and the rapid attenuation [6,7]. As the influence of long-distance propagation in deep rock, microseismic wave is mixed with much noise, which makes it getting quite complicated in signal components. And the noise components in microseismic wave bring difficulties to its process and analysis.

In the field of digital signal process, Fourier transform is the most classical processing method for the stationary signal [8]. It transforms the signal from the time-domain to the frequency-domain, from which two key parameters, vibration peak and basic frequency, can be presented effectively [9]. However, microseismic wave is a typical stochastic signal. Some of its characteristics cannot be roundly given by the Fourier transform. Especially, microseismic signal is non-stationary. It would be contrary to the theoretical basis of Fourier transform if this method is used in the non-stationary signal processing [10]. Thus, J. Morlet, a French geophysicist, firstly proposed the concept of wavelet transform to public in 1984 based on years of researches on oil signal process. Y. Meyer and S. Mallat developed the wavelet theory and subsequently prompted it become an indispensable branch of applied mathematics [11]. Wavelet transform is an important time-frequency analytical method in signal process. It decomposes the signal in both time and frequency domains. It has the favorable property of time-frequency localization, in which a resizable window, fixed in area yet alterable in shape, is applied. For the low-frequency components of signal, a wide time window is used to decrease the time resolution yet increase the frequency resolution, whereas for the high-frequency components of signal, a narrow time window is used to increase the time resolution yet decrease the frequency resolution. In other words, wavelet transform has the higher frequency resolution and the lower time resolution in the low frequency domain, while it has the higher time resolution and the lower frequency resolution in the high frequency domain. Because of this, wavelet transform is also honored as the 'mathematic microscope' and is capable of representing the local property of signal in time and frequency domains [12,13]. Yet, weak frequency resolution in high-frequency domain and weak time resolution in low-frequency domain are the defects of wavelet transform. In practical application, improving the frequency resolution in high-frequency domain of microseismic wave is all the time expected. To address this problem, R. R. Coifman, V. Wickerhauser and Y. Meyer further developed Mallet’s wavelet algorithm and put forward the upgraded concept, WPT, around the year 1990 [14]. The basic idea of WPT is to finely decompose the high-frequency domain of signal as what is done in its low-frequency domain by wavelet transform. By the WPT, the frequency domain of signal is averagely divided in the required decomposing gradation, and the more subtle signal components are obtained. Besides, the time resolution in low-frequency domain of signal is also improved by the WPT. That is, the WPT decomposes any part of the signal with a higher resolution in time and frequency domains, which resolves the conflicts in time resolution and frequency resolution very well. Meanwhile, according to the signal features and analysis requirements, wavelet packet analysis can adaptively select the appropriate frequency band to match the signal frequency spectrum, which is precisely the advantage of WPT [15].

When doing the signal processing using the WPT, an extremely important precondition is the optimal determination of the wavelet packet basis function [16,17]. Just as the effect of the processing method, distinct basis function also leads to the different results. The more ideal results can be obtained based upon the application of the more advanced signal processing technique and the more reasonable basis function. It is generally recognized whether a basis function is an optimum or not determines the impartiality, objectivity and validity of the subsequent signal process [18,19]. Many of wavelet bases have been created since then. Because of their outstanding effects, the basis function in the wavelet families of Haar, Daubechies, Mexihat, Biorthogonal, Morlet, Symlets, Coiflets and Meyer are the most commonly used in the practical engineering. Previous researches have proved that the wavelet bases in Daubechies, Symlets and Coiflets families are more appropriate for the signal processing of earthquake wave (seismic signal) and explosion wave (blast signal) [20,21]. In consideration of the differences in their natural properties, the optimal basis functions that are suitable for the seismic wave and the blast wave cannot be directly applied into the signal processing of underground microseismic wave. Thus, it is necessary to determine an optimal wavelet packet basis function for the microseismic wave in this research. And as the signal process on microseismic wave has just been started in recent years, determination of its optimal wavelet basis is also quite important for the further researches.

2. Wavelet Packet Basis Function and Specialty

2.1 Wavelet function and transform

$\psi(t)$ is a function expressed as $\psi(t) \in L^1(R) \cap L^2(R)$, in which $L^1(R)$ and $L^2(R)$ are the function space consisted of absolutely integrable function and quadratically integrable one respectively. If $\|\psi\|$ meets the standardization condition $\|\psi\| = 1$, and its Fourier transform $\hat{\psi}(\omega)$ meets the following admissible condition,

$$C_\psi = \int_R \frac{\left|\hat{\psi}(\omega)\right|^2}{|\omega|} d\omega < +\infty , \tag{1}$$

then, function $\psi(t)$ is defined as the basic wavelet or mother wavelet. Generally the basic wavelet is a band-pass function in time domain and compactly exists in both time and frequency domain. For any energy-limited function (t) $\left(f(t) \in L^2(R)\right)$, its continuous wavelet transform is defined as

$$WT_f(a,b) = \frac{1}{\sqrt{|a|}} \int_R f(t)\psi^*\left(\frac{t-b}{a}\right) dt = \langle f, \psi_{a,b} \rangle , \tag{2}$$

where, $WT_f(a,b)$ is the coefficient of wavelet transform. $\psi^*(t)$ is the complex conjugate of $\psi(t)$. Parameters a, b and t are continuous variable, which should be discretized as $a = a_0^j (a_0 > 1)$ and $b = k a_0^j b_0 (k \in Z)$ in computerized application. Corresponding reconstructed expression of discrete wavelet transform is

$$\psi_{j,k}(t) = a_0^{j/2} \psi\left(a_0^{-j} t - k b_0\right) . \tag{3}$$

This equation is known as the wavelet base function dependent on scale parameter a and translation parameter b. Discrete changing of these two parameters achieves the discrete wavelet transform. The value of a_0 and b_0 directly impact the precision of signal reconstruction. In order to ensure the signal precision and make wavelet transform adapt to signal nonstationarity, the binary discrete wavelet base ($a_0 = 2,\ b_0 = 1$) is used in this research. The orthogonal wavelet transform is more and more adapted in the discrete wavelet transform, which can greatly cut down the calculated quantity on the premise of intact original signal.

2.2 Analytical approach of wavelet packet transform

In the wavelet multi-resolution analysis, Hilbert space $L^2(R)$ is decomposed as the orthogonal sum of all wavelet subspace $W_j (j \in Z)$, that is $L^2(R) = \oplus_{j \in Z} W_j$, according to different scale 2^j. Subspace W_j is a closure of wavelet function $\psi(t)$. To achieve the purpose of improving frequency resolution, the wavelet subspace W_j should be further decomposed based on the binary mode. Scale subspace V_j and wavelet subspace W_j ($V_{j+1} = V_j + W_j$ in Hilbert space) are represented by a unified space as follows,

$$\left.\begin{aligned} U_j^0 &= V_j \\ U_j^1 &= W_j \end{aligned}\right\} \Longrightarrow U_{j+1}^0 = U_j^0 \oplus U_j^1 \quad j \in Z \,. \tag{4}$$

If U_j^n and U_j^{2n} are the closure space of function $u_n(t)$ and $u_{2n}(t)$, respectively, and they meet the following dual-scale equation,

$$\left.\begin{aligned} u_{2n}(t) &= \sqrt{2}\sum h(k) u_n(2t-k) \\ u_{2n+1}(t) &= \sqrt{2}\sum g(k) u_n(2t-k) \end{aligned}\right\} \quad k \in Z, n = 1,2,3,\cdots, \tag{5}$$

then,

$$U_{j+1}^n = U_j^{2n} \oplus U_j^{2n+1} \,. \tag{6}$$

Based upon above relation, the orthogonal wavelet packet determined by base function $u_0(t)$ is defined as:

$$\left.\begin{aligned} W_j &= U_{j-1}^2 \oplus U_{j-1}^3 \\ W_j &= U_{j-2}^4 \oplus U_{j-2}^5 \oplus U_{j-2}^6 \oplus U_{j-2}^7 \\ &\vdots \\ W_j &= U_{j-k}^{2^k} \oplus U_{j-k}^{2^k+1} \oplus \cdots \oplus U_{j-k}^{2^{k+1}-2} \oplus U_{j-k}^{2^{k+1}-1} \\ &\vdots \\ W_j &= U_0^{2^k} \oplus U_0^{2^k+1} \oplus \cdots \oplus U_0^{2^{k+1}-1} \end{aligned}\right\}, \tag{7}$$

where, the function set $2^{(j-k)/2} u_{2^k+m}(2^{j-k}t - l)$ $(l \in Z, m = 0,1,2,\cdots,2^k - 1)$ is a orthonormal basis of $U_{j-k}^{2^k+m}$. Wavelet packet function can be briefly expressed as $\psi_{j,k,n}(t)$ $(n = 2^k + m)$, in which parameters *j*, *k*, *n* represent the scale, displacement and frequency indicators, respectively. Compared to the wavelet function $\psi_{j,k}(t)$, parameter *n* as the frequency representation in wavelet packet function overcomes a defect of poor frequency resolution in high frequency range of signal. The following **Fig. 1** shows the wavelet packet decomposition in the third layer. For a specific signal, 2^n frequency sub-bands can be obtained in the *n*th decomposition layer. The original signal can be completely reconstructed by these aequilate sub-bands. If the maximum frequency of original signal is *f*, the bandwidth of each frequency sub-band will be $f/2^n$. Based on the deep signal decomposition, the signal components in different frequency bands will be clearly identified and analyzed in detail. In application, the signal decomposing gradation is decided based upon the specific requirements.

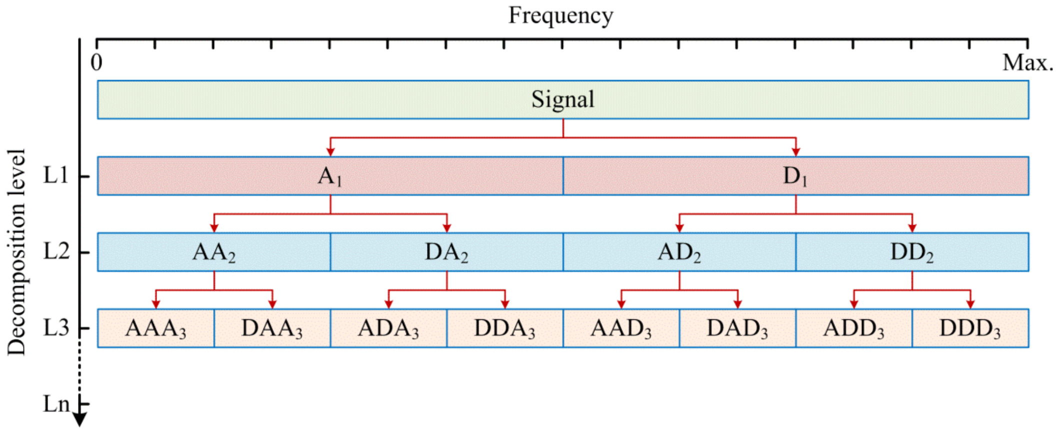

Fig. 1 Decomposition structure in frequency domain of signal by the WPT. In the filtering process at the basic level, the original signal, S, passes through two complementary filters (low-pass & high-pass) and emerges as two signals A_1 and D_1. Multi-level signal decomposition follows the some rules.

2.3 Specialty of wavelet basis function

Compared to the standard Fourier transform, the wavelet basis functions are non-unique. All of the orthogonal, non-orthogonal, biorthogonal and linearly-dependent wavelet bases can be applied into the microseismic wave processing [22,23]. However, different results are obtained spontaneously. To achieve the optimal determination, the basis function should be screened preliminarily based on the characteristics of the wavelets and the characteristics of the microseismic wave. Respective mathematical characteristics of the common wavelet packet bases are listed in the **Table 1**.

Table 1 Characteristics of the basis functions in each wavelet family [24].

	Families of the wavelet packet basis functions							
	Haar	Daubechies	Biorthogonal	Coiflets	Symlets	Morlet	Mexihat	Meyer
Form of expression	Haar	dbN	biorNr.Nd	coifN	symN	morl	mexh	meyr
Orthogonality	Y	Y	N	Y	Y	N	N	Y
Biorthogonality	Y	Y	Y	Y	Y	N	N	Y
Compact support	Y	Y	Y	Y	Y	N	N	N
Support width	1	2N-1	reconstruction: 2Nr+1 decomposition: 2Nd+1	6N-1	2N-1	finite	finite	finite
Filter length	2	2N	Max: (2Nr,2Nd)+2	6N	2N	[-4, 4]	[-5, 5]	[-8, 8]
Symmetry	Y	approximate	N	approximate	approximate	Y	Y	Y
Order of vanishing moment	1	N	Nr-1	2N	N	N	N	N

* Y means the wavelet basis in relevant family has the corresponding specialty, whereas N is the opposite.

High characteristics of compact support, smoothness and symmetry of the wavelet packet bases are required for the signal reconfiguration of microseismic wave. Compact support can insure the excellent local features of the discrete orthogonal wavelet packet bases in space; the smoothness can result in the superior frequency resolution, and the symmetry can assure that wavelet bases have the filter characteristics in transformation, which can improve the signal fidelity. However, these three features usually cannot be met simultaneously in the WPT. Especially, the compact support and smoothness cannot be held at the same time. These characteristics lead to their distinct signal reconstruction capability and adaptability, and should be taken into consideration as carefully and comprehensively as possible in application. Based on the characteristics of wavelet bases and the specialties of microseismic wave, the basis functions in Daubechies, Symlets and Coiflets families are preliminarily determined as the more appropriate wavelet bases for the microseismic wave.

3. Case Study and Results

3.1 Evaluation parameters for the optimal selection of wavelet basis

Whether a wavelet packet basis function is optimum or not for the microseismic wave is mainly determined by its reconstruction capability on the original signal and the conservation capability on the characteristic information of original signal (characteristic signal). The characteristic signal is obtained by reconstructing the coefficients of sampling data in the corresponding frequency domain. The optimal wavelet basis should have a better even the best capability to reconstruct the original wave and to retain the characteristic signal as much as possible [25,26]. These two capabilities are evaluated by two parameters, the root mean square error (RMSE) of the reconstructed signal and the original signal, and the correlation coefficient (CC) of the characteristic signal and the original signal. RMSE value reflects the capability of reconstructing a signal using a wavelet basis, and is used for selecting the more appropriate basis function. The CC value implies the accuracy of the characteristic signal extracted from the original one by a specific wavelet, and helps to determine the optimal basis function. Expressions of the RMSE and the CC are defines as:

$$E_{RMSE} = \left[\sum_{i=1}^{N} \left| S_r(i) - S(i) \right|^2 \Big/ N \right]^{1/2} , \tag{8}$$

$$r_{CC} = \sum_{i=1}^{N} S(i)\bar{S}(i) \Bigg/ \left[\sum_{i=1}^{N} S^2(i) \sum_{i=1}^{N} \bar{S}^2(i) \right]^{1/2} , \tag{9}$$

where, E_{RMSE} is the RMSE value and r_{CC} is the CC value. N is the number of the sampling data in signal, and S, S_r and $\bar{S}$ are the original signal, reconstructed signal and characteristic signal, respectively. In general, the less the RMSE value is, the better the reconstructed signal is; and the greater the CC value is, the higher the accuracy of the reconstructed signal is.

3.2 Sampling wave group

To specify the optimal determination of the wavelet basis, a microseismic wave group is randomly selected from the data set and is used as the discussion object. The sampling rate of the checked wave object is 500Hz. Their waveforms are shown in the **Fig. 2** and the corresponding location information is listed in the **Table 2**.

Figure 2 shows that, with the increase of propagation distance, microseismic waves hold the longer time duration and the lesser particle velocity. Arrival time of the P-wave is difficult to identify as the decrease of particle velocity and the arrival time difference between P-wave and S-wave becomes larger. When detected at almost the same distance, both waves are also greatly differing with each other in waveform and components as the distinction of propagation paths. It indicates that the selection of an optimal wavelet basis for the signal reconstruction and de-noise is particularly important.

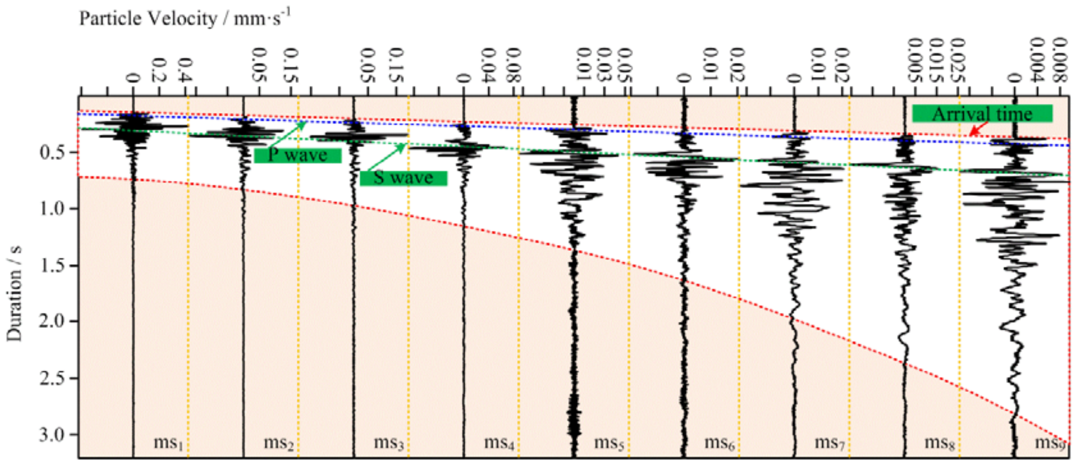

Fig. 2 A typical microseismic wave group sorted upon the arrival time from early to late. These wave data was released from the same seismic event but monitored by nine wave detectors located in separate positions of underground deep rock environment. The length of wave propagation path changes from hundred meters to thousand meters. The fracture development degree of rock mass on these paths is quite high.

Table 2 Location information of the wave detectors and Microseismic focus.

Microseismic wave	Wave detectors		Microseismic focus
	distance (m)	depth (m)	depth (m)
ms1	314.33	-858.2	-911.1
ms2	449.05	-824.7	
ms3	569.35	-885.2	
ms4	774.24	-959.1	
ms5	822.21	-717.6	
ms6	1115.80	-1076.1	
ms7	1064.03	-709.0	
ms8	1134.87	-1025.3	
ms9	1335.49	-672.0	

3.3 Comparison of signal reconstruction capability

Two screening approaches are always succinctly used for the optimal selection in some studies. One is the comparative analysis to the basis function in the same family but with the different filter length; another is the comparative analysis to the basis function with the same filter length but in the disparate family [27,28]. Although these two approaches reduce the workload and save time effectively, they are not all-inclusive as much as possible. In other words, the wavelet bases cannot be compared comprehensively. So, in order to make the optimal selection more convictive, all of the wavelet bases in the Daubechies, Symlets and Coiflets families were compared in this study. Any discussion of wavelets begins with Haar wavelet, the first and simplest. Haar wavelet is discontinuous, and resembles a step function. It represents the same wavelet as Daubechies db1. I. Daubechies, one of the brightest stars in the world of wavelet research, invented what are called compactly supported orthonormal wavelets with extremal phase and highest number of vanishing moments for a given support width — thus making discrete wavelet analysis practicable. Associated scaling filters are minimum-phase filters. The Symlets are nearly symmetrical wavelets proposed by Daubechies as modifications to the db family. The properties of the two wavelet families are similar. Symlets wavelets are also compactly supported wavelets with least asymmetry and highest number of vanishing moments for a given support width. Wavelets in Coiflets family are also built by I. Daubechies at the request of R. Coifman. The wavelet function has 2*N* moments equal to 0 and the scaling function has 2*N*-1 moments equal to 0. The two functions have a support of length 6*N*-1. Coiflets wavelets are also compactly supported wavelets with highest number of vanishing moments for both *phi* and *psi* for a given support width. Wavelet functions of members of these three families are shown in **Fig. 3**.

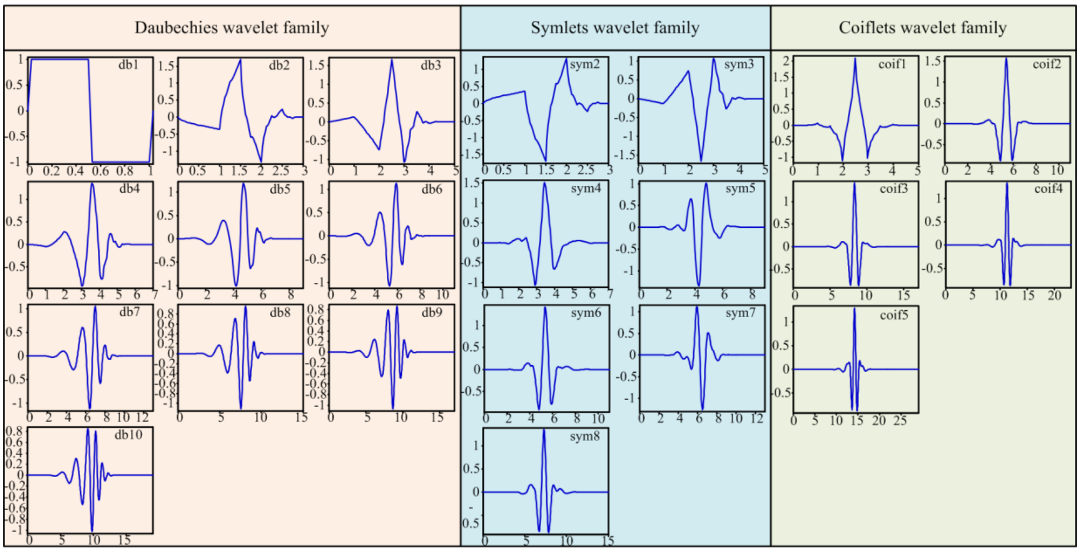

Fig. 3 Wavelet basis functions of members in Daubechies, Symlets and Coiflets families.

After the decomposition and reconstruction processes of waves, the RMSE results of each microseismic wave decomposed by the different bases in different level (frequently-used level 1-8) are shown in the **Fig. 4**.

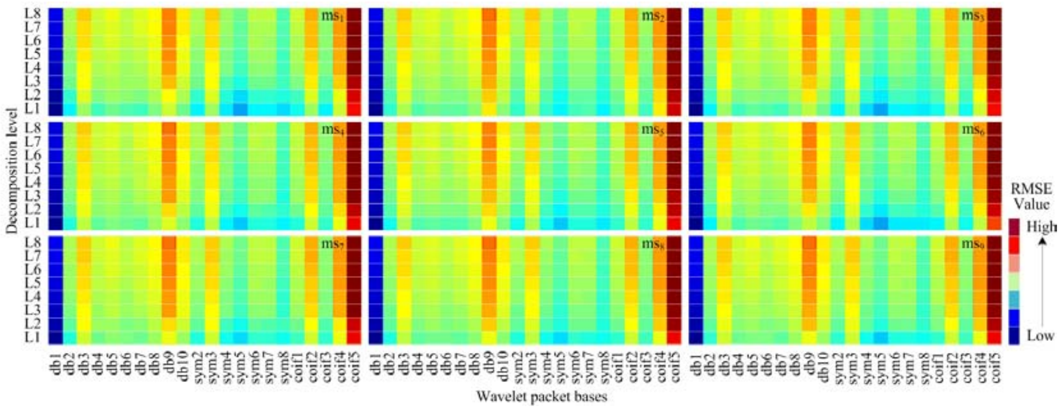

Fig. 4 RMSE phase spectrum of the microseismic waves at different decomposition level.

Results indicate that the RMSE value generated from the Coiflets bases is averagely greater than that calculated from the Daubechies and Symlets bases. At the same decomposition level, RMSE value reaches to the minimum when the waves are reconstructed by the basis db1; and when decomposed by the same wavelet bases, the RMSE value also reaches to the minimum at the decomposition level 1. The RMSE phase spectrum indicates that only the results in the decomposition level 1 is adequate for the optimal determination. Thus, the RMSE results of waves in the decomposition level 1 are compared as shown in the **Fig. 5**.

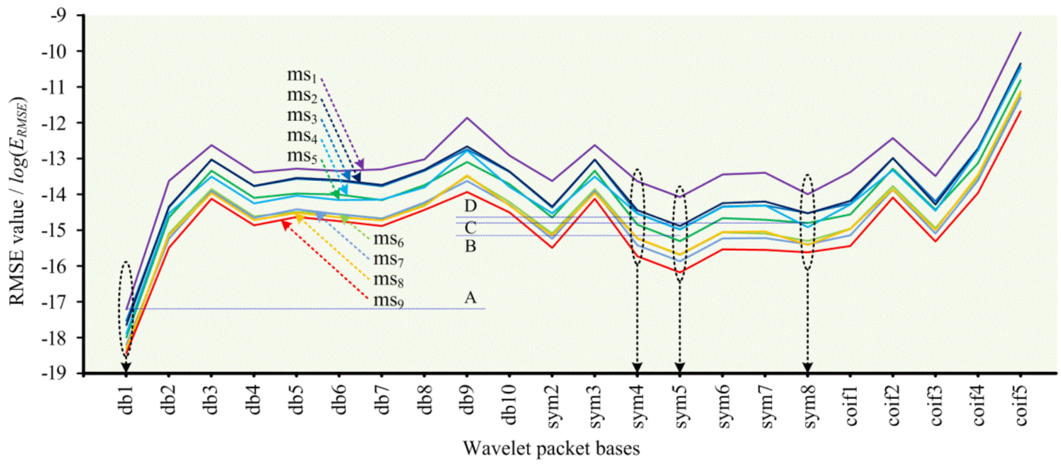

Fig. 5 Comparison of the RMSE results at the decomposition level 1.

It indicates that the order of magnitude of the maximum RMSE is basically lower than 1.0×10^{-13}. In general, the greater the wave propagation distance is, the less the RMSE value is. The wavelet basis db1 has a better signal reconstruction capability for the remote wave. Secondary minimum RMSE value emerges when the waves are reconstructed by the bases sym5, sym8 and sym4. Thus, it indicates that the wavelet bases db1, sym5, sym8 and sym4 have the better signal reconstruction capability for microseismic waves, and the corresponding capability decreases in turn.

3.4 Determination of optimal wavelet basis based on the conservation capability of characteristic signal

As mentioned above, the conservation capability of characteristic signal can be assessed by the CC value that helps to the final determination of the optimal wavelet basis. Thus, the CC results of original signal and reconstructed one calculated based upon these four bases are listed in the **Table 3**.

Table 3 CC results of each wavelet basis in transformation.

Microseismic waves	Wavelet packet bases			
	db1	sym4	sym5	sym8
ms1	0.9116	0.9475	0.9606	0.9601
ms2	0.9703	0.9945	0.9942	0.9961
ms3	0.974	0.9976	0.9986	0.9981
ms4	0.9677	0.9964	0.9965	0.9983
ms5	0.9805	0.9862	0.9862	0.986
ms6	0.9662	0.986	0.9879	0.9908
ms7	0.9828	0.9959	0.9974	0.9968
ms8	0.9913	0.9989	0.9992	0.9992
ms9	0.9718	0.9933	0.9956	0.9949
Priority	D	C	A	B

CC results indicate that it is completely practicable when the microseismic waves are decomposed by the bases db1, sym4, sym5 and sym8. However, the wave reconstructed by basis sym5 has the highest relevancy with the original wave. Basis db1 has the worst conservation capability for the characteristic signal, which is presumably related with the lower signal sampling frequency of the monitoring instrument. Bases sym5 and sym8 balance the requirements quite well. When the priority of RMSE and CC results of these basis functions is endowed with distinct weight value, the average weight of each wavelet basis is then listed in the **Table 4**.

Table 4 Average weight of the alternative wavelet bases.

	db1	sym4	sym5	sym8
RMSE results	4	1	3	2
CC results	1	2	4	3
Average weight	2.5	1.5	3.5	2.5

* Weight value: A — 4, B — 3, C — 2, D — 1.

Based upon the global weight value of each basis function and the screening principle, it can be found that sym5 is the optimal wavelet basis function for the specific microseismic waves. The optimal basis function should be given the highest priority in application. Using the optimal basis sym5, signal error of the reconstructed microseismic wave group is shown in the **Fig. 6**. It indicates that the magnitude of reconstruction error is quite small (lower than 1.0×10^{-13}). That is, the reconstructed wave is almost exactly consistent with the original wave. Thus, it is entirely feasible for the microseismic wave to be decomposed and reconstructed using basis sym5 in the WPT.

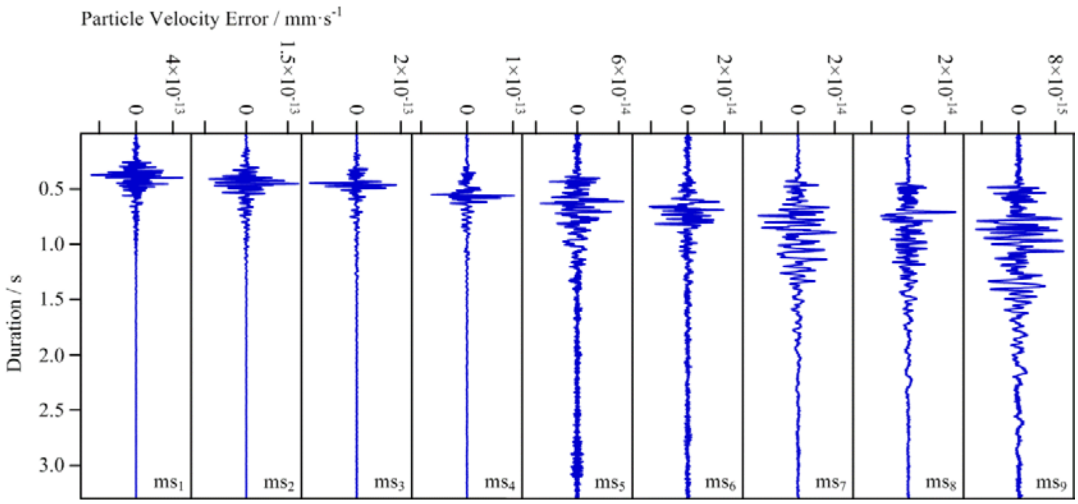

Fig. 6 Reconstruction error of the microseismic waves.

4. Discussion on Energy Reserving Capability of the Optimal Wavelet Basis

4.1 Energy representation of wavelet packet basis

Wavelet packet basis function is briefly expressed as $\psi_{j,k,n}(t)$ ($n=2^k+m$) in which parameters j, k, n represent the scale, displacement and frequency respectively. The frequency representation n overcomes the defect of wavelet transform that is the poor resolution in the high frequency domain of signal. After the decomposition of wave $s(t)$ in WPT, following expression is defined at a specific decomposing level i,

$$s(t)=\sum_{j=0}^{2^i-1} f_{i,j}(t_j)= f_{i,0}(t_0)+\ldots+ f_{i,2^i-1}(t_{2^i-1}) \quad , \tag{10}$$

where, $f_{i,j}(t_j)$ (i = 1, 2, 3, ···; j = 0, 1, 2, ···, 2^i-1) is the reconstructed wave in the decomposition level i and frequency bandwidth $f_{max}/2^i$. Based upon the Parseval theorem [26,27], wavelet packet energy in each decomposing frequency band is calculated by

$$E_{i,j} = \int_T \left| f_{i,j}\left(t_j\right) \right|^2 dt \quad , \tag{11}$$

and total energy is defined as

$$E = \sum_{j=0}^{2^i-1} E_{i,j} = \sum_{0}^{2^i-1} \left(\int_T \left| f_{i,j}\left(t_j\right) \right|^2 dt \right) \quad . \tag{12}$$

4.2 Energy reserving capability of the optimal basis

Conservation capability of the wavelet basis function on the characteristic information of the microseismic wave is mainly reflected in its reserving capability on the energy components of signal. To further verify the superiority of the optimal wavelet basis, the energy reserving capability of basis sym5 and the subordinate bases db1 and sym8 is compared in the frequency-energy changing curves, which is as shown in the **Fig. 7**.

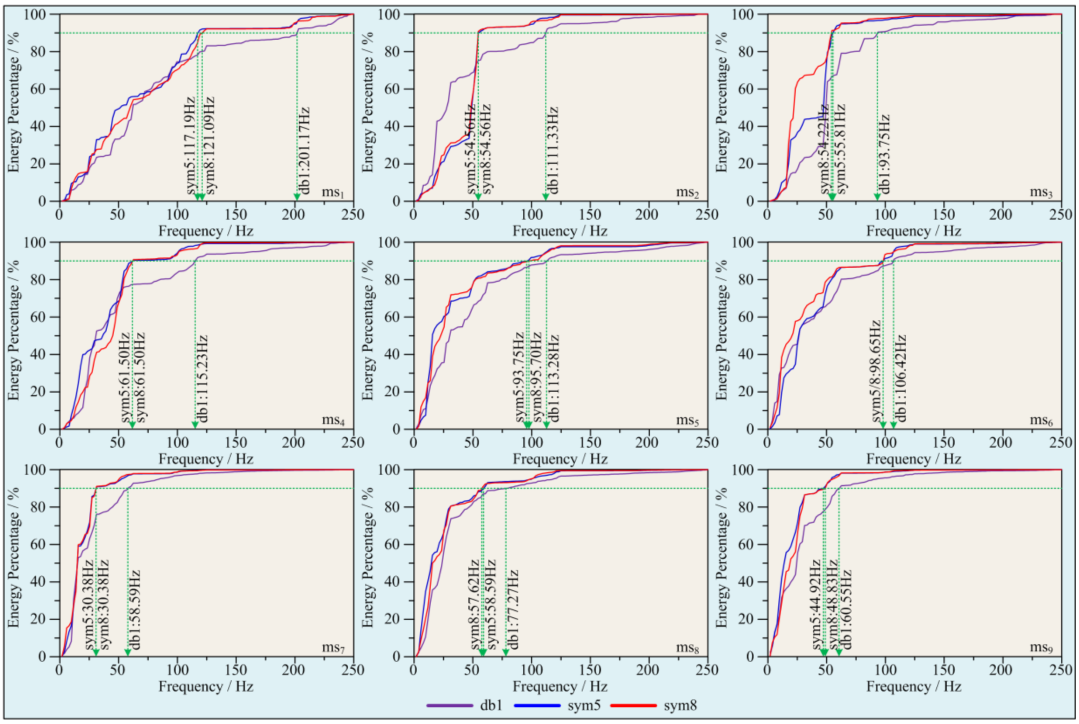

Fig. 7 Frequency-energy changing trends of the waves when transformed by bases sym5, db1 and sym8.

It indicates that the wavelet bases sym5 and sym8 have a higher energy reserving capability than basis db1 in the same frequency domain. To retain the same extent of energy components in waves (90% energy retaining rate is chosen as a criterion in the above figures), the lesser frequency band is adequate for the wavelets sym5 and sym8, whereas a broader frequency band is necessary for the basis db1. The frequency scale is unequal when energy retaining ratio reaches the same threshold. Noise components of the wave are filtered as many as possible, whereas the energy components are furthest reserved [28]. Only the sampling data in half of Nyquist frequency domain is adequate to achieve an effective reservation for the energy components. Frequency-energy correlation of each wave is a little different when the waves are transformed by different bases. But it is as close to the truth as possible. Meanwhile, the high energy components of waves mainly concentrate on the low frequency band. This frequency bandwidth becomes lower and narrower as the wave propagation distance increases.

Under the same energy reserving threshold, it is theoretically thought that the more energy components should be retained when a wave is decomposed and reconstructed by the basis db1 as its broader reserved frequency domain. That is, compared to the bases sym5 and sym8, the wave reconstructed by the wavelet db1 should have the higher relevancy with its original wave. However, CC results of the compressed signal and the original signal listed in the **Table 5** deny this point apparently. It indicates that although the less energy components are retained, reconstructed wave still has a high relevancy with its original wave when the wave is transformed by bases sym5 and sym8. The relevancy is even greater than that induced from the wavelet db1.

Table 5 Correlation coefficient results of the compressed waves and their original.

Microseismic waves	Wavelet packet bases		
	db1	sym5	sym8
ms1	0.9484	0.9491	0.9488
ms2	0.9482	0.9492	0.9479
ms3	0.9482	0.9495	0.9485
ms4	0.9483	0.9493	0.9494
ms5	0.9488	0.9618	0.9546
ms6	0.9490	0.9612	0.9544
ms7	0.9491	0.9497	0.9499
ms8	0.9489	0.9490	0.9492
ms9	0.9486	0.9507	0.9495

The compact support and the vanishing moment of wavelet bases increase steadily with the increase in filter length, which ensures the smoothness but lowers the local quality of waves. CC results calculated from the wavelet sym8 are approximate with that of basis sym5. However, the filter length of basis sym8 is a little longer than that of the basis sym5. In this case, the local quality of the microseismic wave will be decreased slightly during the signal reconstruction by the wavelet sym8. Thus, on this view, wavelet sym5 is the optimal basis function for the WPT of microseismic waves. Generally, the wavelets in Symlets family are improved basis functions of the Daubechies family. Their waveforms promote them to be more suitable for reconstructing the mining-induced microseismic wave. In the same support domain, the smoothness of Symlets bases is always better than that of the Daubechies bases. These specialties make the Symlets bases more suitable for the signal process and analysis of the microseismic waves.

5. Application of the Optimal Wavelet Basis on the Wave Denoising Process

After the optimal wavelet basis is determined, the next consideration is just its application on the denoising process of the initial microseismic waves. During the monitoring process, the measured wave data is inevitably mixed with the noise components as the interference induced from the geomagnetic field, electric discharge and machinery operation [29]. This interference decreases the accuracy and increases the ambiguity of the microseismic waves. Noise components play an adverse impact on wave analysis. Thus, denoising process appears particularly important to improve the wave reliability. In signal processing, denoising action to the initial signal is an important issue. Main purpose of the signal denoising process is filtering the high-frequency low-energy noise components but reserving the effective low-frequency high-energy wave components. As the microseismic wave is one kind of the typical non-stationary signal, wavelet packet coefficients of the effective wave components and the random noise components present different characteristics. With the increase of the decomposition level, wavelet packet coefficients of the pure noise get smaller and smaller, whereas the wavelet packet coefficients of the effective signal get more and more obvious. Thus, the denoising process of the original microseismic wave can be achieved if only the wavelet packet coefficients of the effective signal are reserved. The maximal modulus value, the threshold value and the invariant translation are three commonly-used denoising methods in the current signal process.

6. Conclusions

Following conclusions can be made based on the above results and discussions:

(1) Different wavelet basis has the different signal reconstruction capability and adaptability. The features of wavelet bases and the specialties of microseismic waves suggest that the basis function in Daubechies, Symlets and Coiflets families is more appropriate for the signal analysis and process of microseismic waves. Signal reconstruction capability generated from the Coiflets bases is averagely less than that reflected by the Daubechies and Symlets bases. Compared with other wavelet bases, basesdb1, sym4, sym5 and sym8 have the better reconstruction capability for the microseismic waves. Magnitude of the reconstruction error is quite infinitesimal, which is generally lower than 10^{-13}.

(2) Among these four bases, db1 has the worst conservation capability for the characteristic signal, whereas the wave reconstructed by the sym5 has the highest relevancy with the original wave, and the maximum energy components can be effectively reserved. The longer the wave propagation distance is, the easier the energy concentration in low frequency domain is, and the noise components get greater and concentrate on the high frequency domain. Based upon the RMSE results, CC results and the comparison of energy reserving capability, the basis function sym5 is proved to be the optimal wavelet packet basis for the WPT of microseismic waves, and should be given the highest priority in application.

(3) Application of the results is mainly related to the decomposition, reconstruction, denoising and filtering of the initial microseismic waves. After the wave processing stage, the purified reconstructed waves can be applied into the further analysis and practice such as the spectral analysis, energy identification, wave energy attenuation, focus energy inversion, spatial attenuation zoning, hazard regional division, and other fields. Just for these reasons, the optimal determination of a wavelet basis function becomes quite important for the further researches.

Acknowledgements

The support for this study, general and financial, provided by the G-COE Program in Novel Carbon Resource Sciences of Kyushu University is gratefully acknowledged.

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