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A Proposal of Harmonictron

by

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Abstract

The possibility of high intensity hadron accelerator based on a vertical scaling FFAG with harmonic number jump acceleration, named “Harmonictron”, is proposed. The paper also gives a design example of the Harmonictron for accelerating protons from 50MeV to 500MeV, which could be used for generating various secondary particles such as pion, neutron, etc.

Keywords: Vertical scaling FFAG, Harmonic number jump, Harmonictron

1. Introduction

Demands for high intensity and high energy hadron accelerators are strongly increasing in various fields recently. Especially, to generate intense secondary particles such as pion, muon and neutron, a high intensity accelerator which is able to accelerate the protons or deuterons up to the energy of 500MeV/u-1GeV/u has been requested. Both linear and circular accelerators such as cyclotrons or synchrotrons have reached a beam power (beam energy × averaged beam current) of about 1MW level. However, for reaching more intense beam power of more than 10MW, ring accelerators such as cyclotrons and synchrotrons seem to be difficult. The synchrotron is a pulse operated accelerator where the averaged beam current can be limited by the beam duty factor in operation where the field strength of the magnets must be time-dependent to keep the circular orbit radius constant during beam acceleration which limits the duty factor in operation. For example, the J-PARC 3GeV rapid cycling synchrotron (RCS) limits the repetition rate to 25Hz in ordinary operation.

Fixed magnetic field ring accelerators such as cyclotrons or fixed field alternating gradient (FFAG) accelerators[1)] have capabilities for high intensity beam acceleration. In the cyclotron, however, it has been recognized that the beam bunch shape is deteriorated by the space charge force and the beam extraction becomes difficult when the beam current exceeds 5mA. In FFAG accelerators, either scaling or non-scaling, a continuous wave (cw) operation with radio frequency

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(rf) field has some difficulties for accelerating non-relativistic energy heavy particles such as the proton. The revolution period for each turn varies for non-relativistic energy particles, thus, the rf frequency should also be varied to synchronize the particle velocity. Thus, the fixed frequency rf acceleration without losing synchronization is essential.

To overcome these difficulties, an idea of harmonic number jump (HNJ) acceleration has been proposed by by Ruggiero[2]. And, it has been demonstrated that the HNJ scheme is useful for accelerating high energy (relativistic) muons with the scaling FFAG accelerator[3),4)].

The condition of synchronization in the circular accelerator between the revolution periods for each turn, $T(E_i)$, where E_i is a particle energy of i-th turn, and the frequency of rf field, f_{rf}, is given by,

$$f_{rf} = \frac{h_i}{T(E_i)} \quad . \tag{1}$$

Here, h_i is an integer number called the harmonic number. To maintain the above condition in a non-isochronous ($T(E_i) \neq$ constant) ring with fixed rf frequency, the harmonic number h_i should be changed by an integer number for every turn.

The scheme of HNJ acceleration, however, has some difficulties to accelerate heavy particles such as protons or deuterons for a wide range of medium (non-relativistic) energy. In ordinary strong (AG) focusing circular accelerators, the transition energy exists inevitably during beam acceleration without making a special arrangement of beam optics which is rather difficult in the scaling FFAG accelerator.

At the transition energy, many problems occur for leading beam losses in high intensity beam acceleration caused by collective beam instabilities such as negative mass instability when the beam crosses the transition energy. In HNJ acceleration, a sign of harmonic number increment changes before and after the transition energy; the harmonic number decreases before and increases after the transition energy. If the transition energy is eliminated in the scaling FFAG accelerator, the HNJ acceleration could become more useful and efficient for high intensity proton and heavy ion accelerators.

In order to eliminate the transition energy inherently, momentum compaction in beam dynamics must be zero like linear accelerator and ordinary ring accelerator is not. In a vertical scaling FFAG accelerator, as its unique feature despite the ring accelerator, momentum compaction is zero because of a constant orbit radius whatever the beam energy.

The idea of the vertical scaling FFAG accelerator was originally proposed by Ohkawa in 1955[5)] and analyzed in detail by Brooks recently[6)]. The vertical scaling FFAG accelerator has a unique feature where the beam moves vertically and its orbit radius is always constant during acceleration. Thus, constant rf frequency acceleration for relativistic particles such as the electron becomes possible and Ohkawa named a vertical scaling FFAG accelerator “electron cyclotron”. The feature that orbit radius is always constant, in other words, means the zero-momentum compaction and no transition energy exists like linear accelerator which is suitable for applying the HNJ acceleration.

We propose in this paper, a new scheme of fixed field and cw operation accelerator with HNJ acceleration using a vertical scaling FFAG concept to eliminate the transition energy, which is called “Harmonictron”.

2. Scheme of Harmonictron

2.1 Outline of vertical scaling FFAG accelerator

In the vertical scaling FFAG accelerator, the magnetic field strength changes exponentially in the vertical direction to keep a zero chromatic beam optics with constant orbit radius. In the relativistic energy range, where particle velocity almost equals light velocity, a light mass particle such as the electron can be accelerated with constant frequency rf field in the vertical scaling FFAG accelerator. Thus, Ohkawa named it "electron cyclotron". Of course, even non-relativistic particle such as proton can be accelerated by the rf field which synchronizes a time revolution elapsing around the ring for each turn as in the ordinary proton synchrotron.

In the vertical FFAG accelerator, the magnetic field strength changes vertically as,

$$B_y = B_0 \exp(my) \quad . \tag{2}$$

Here, y is a vertical position in x-y coordinates and a characteristic number m is expressed as,

$$m = n/\rho \quad , \tag{3}$$

where ρ is an orbit curvature and n is a field index defined by,

$$n = \frac{\rho}{B_y}\frac{dB_y}{dy} \quad . \tag{4}$$

The linearized particle motion in the transverse direction which is subject to a skew quadrupole magnetic field presented in eq.(4) can be expressed by the following betatron equation in skew coordinates (u_+, u_-) under the approximation of no orbit curvature effect ($\rho\rightarrow$large).

$$\mathbf{z}_\pm'' + k_\pm \mathbf{z}_\pm = 0 \quad , \tag{5}$$

where

$$\mathbf{z}_\pm = \begin{bmatrix} u_+ \\ u_- \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad , \tag{6}$$

(x, y) is a position of a particle in x-y coordinates.
Here,

$$\mathbf{z}_\pm'' = d^2\mathbf{z}_\pm / ds^2 \quad (s\text{: flight path length}) ,$$

and

$$k_\pm = \mp(m/\rho) \quad . \tag{7}$$

Since the beam orbit curvature, ρ, is constant whatever the particle momentum, k becomes constant and the betatron tunes keep constant, which complies with zero chromaticity.

The characteristic number of m specifies the orbit displacement, y_d, between initial momentum (p_i) and final beam momentum (p_e) as $m = (1/y_d)\ln(p_e/p_i)$. If p_e/p_i equals 3 and y_d is less than 2m, then, m should be more than 0.55m^{-1}. In **Table 1**, the typical machine parameters of a vertical scaling FFAG accelerator which accelerates proton from 50MeV to 500MeV is presented.

One of the features of the vertical scaling FFAG accelerator is that the momentum compaction is zero because the orbit radius (curvature) keeps constant whatever the beam momentum. This plays an important role for HNJ acceleration in the vertical scaling FFAG as described later.

Table 1 The beam parameters.

Particle	proton
Injection energy	50MeV
Extraction energy	500MeV
Ring circumference	64m
Maximum B field	2.5T
Characteristic number	1.00m^{-1}
Number of cells	16
Orbit elevation	1.23m
Beam orbit displacement at maximum energy	0.11m
Initial harmonic number	80
Change of harmonics/turn	-1
rf frequency	117.8MHz

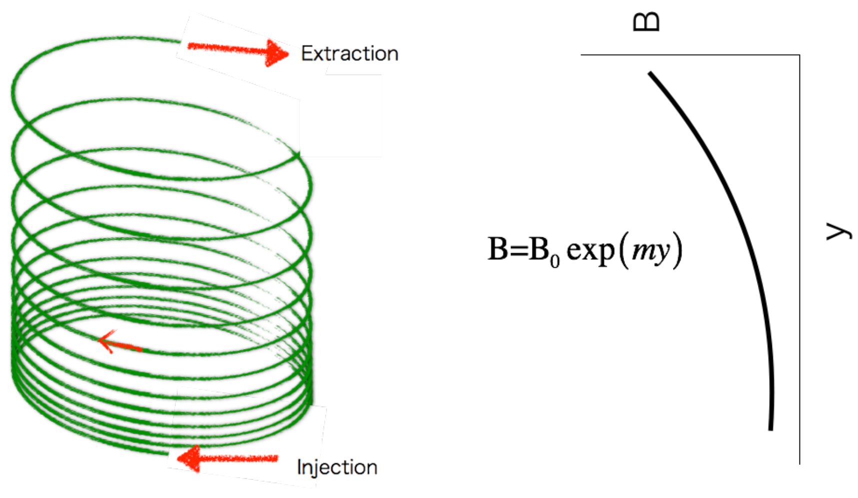


Fig. 1 Schematic beam trajectory (left) and magnetic field strength as a function of vertical axis (right) of vertical FFAG accelerator.

2.2 HNJ acceleration in vertical scaling FFAG accelerator

From the synchronization condition of HNJ acceleration given by eq.(1), the required energy gain to jump an integer harmonic number $\Delta_i h$ of harmonics between the turns *i* and *i*+1 can be expressed with as[3],

$$E_{i+1}\text{-}E_i=\Delta_i h \Big/ \left[f_{rf} \cdot \left(\frac{\partial T}{\partial E}\right)_{E=E_i} \right] \quad . \tag{8}$$

Here, *T* is a revolution time of piecewise linearized around the particle energy.

In ordinary circular accelerators, the relative change in revolution period of a particle with momentum can be expressed with a slippage factor defined as,

$$\eta = \alpha_p - \frac{1}{\gamma^2} \quad . \tag{9}$$

Here α_p is a momentum compaction factor and γ is a Lorentz factor.

The particle energy where η=0 is a transition energy, which can be given by,

$$\gamma_t = \frac{1}{\sqrt{\alpha_p}} \quad . \tag{10}$$

Below the transition energy, η<0, then,

$$\frac{\partial T}{\partial E} < 0 \quad . \tag{11}$$

Thus, harmonic number decreases during beam acceleration. On the other hand, the harmonic number increases above the transition energy.

The rf phase synchronizing the particle motion is normally less than π below the transition energy and more than π above the transition energy to keep a stable beam acceleration. Therefore, a fast rf phase jump becomes essential in beam acceleration to cross the transition energy. The horizontal scaling FFAG accelerator, whose momentum compaction is positive, has difficulty to avoid the transition in a wide energy range acceleration of heavy particles such as proton.

In the vertical scaling FFAG accelerator, the momentum compaction becomes always zero because the orbit radius is constant during acceleration. Thus, the transition energy is infinite, in other words, no transition energy exists, and the beam is accelerated always below transition in the vertical scaling FFAG so that many problems caused by the transition energy described above can be avoided. Applying HNJ acceleration to the vertical scaling FFAG, heavy particles such as the proton can be accelerated in wide range of the non-relativistic energies with cw operation mode like linear accelerator. We name this new type of accelerator based on vertical scaling FFAG with HNJ acceleration as “Harmonictron”.

The term dT/dE of required energy gain in eq.(8) can be expressed in the following equation with a slippage factor given by eq.(9).

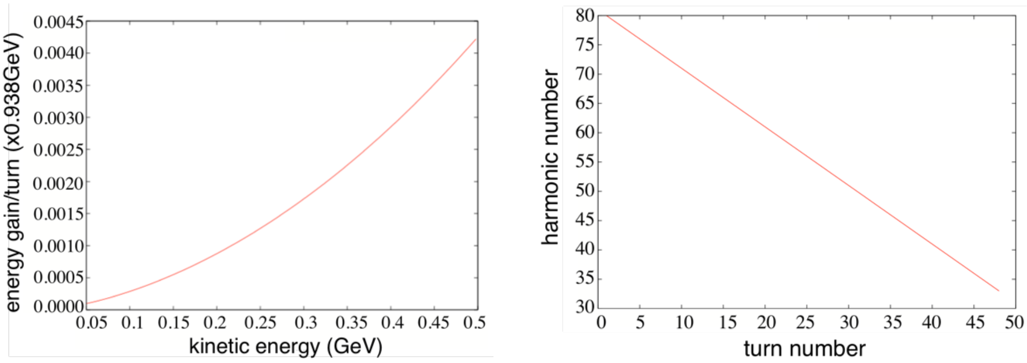
$$\frac{dT}{dE} = \frac{\eta\gamma^2 C}{M_0 c\left(\gamma^2 - 1\right)^{3/2}} \quad , \tag{12}$$

where C is the circumference of the ring, c is light velocity and M_0 is rest mass energy (M_0=938.2MeV for proton). Since the momentum compaction is zero in Harmonictron (vertical scaling FFAG), eq.(8) can be expressed as,

$$E_{i+1} - E_i = -\frac{\Delta_i h M_0 \left(\gamma_i^2 - 1\right)^{3/2}}{f_{rf}} \frac{c}{C} . \tag{13}$$

As can be seen from this equation, the required energy gain per turn is a function of γ_i since f_{rf} is constant and $\Delta_i h$ should be a negative value for acceleration.

Figure 2(a) shows the energy gain per turn given by eq.(13) for the case whose beam parameters are shown in **Table 1** and a change of harmonic number per turn ($\Delta_i h$) equals to -1. In this case, the harmonic number is 80 at the injection energy of 50MeV and decreases to 47 at the maximum energy of 509MeV, which is shown in **Fig.2(b)**.



(a) (b)

Fig.2 Energy gain per turn as a function of kinetic energy(a) and harmonic number as a function of turn number(b).

As can be seen from **Fig.2(a)**, the energy gain required for each turn in HNJ acceleration increases largely according to the beam energy. The energy gain for each turn is given by an rf voltage, V_i and phase, Φ_i as shown in the following equation.

$$E_{i+1} - E_i = QeV_i \sin \Phi_i , \tag{14}$$

where Qe is an electric charge of a particle. Here, either V_i or Φ_i should be varied to match the condition of energy gain shown in eq.(16). **Figure 3** shows the energy gain per turn as a function of turn numbers.

Figure 4 shows the change of required rf voltage, V_i, as a function of the turn numbers when the stable phase Φ_i is constant with 60°. **Figure 5** shows the change of stable phase, Φ_i, when the rf voltage V_i is constant with 50MV.

The energy gain increases largely at the end of beam acceleration so that the beam orbit turn separation may become largest at the final turn. This is a good feature for beam extraction, especially in cw beam operation, which may not need any special devices for beam extraction. For example, if m=1.00 m^{-1} as shown in **Table 1**, the orbit turn separation at the final turn is about 11cm, which seems to be large enough for beam extraction.

As can be seen from **Fig.4** and **5**, the rf voltage and/or phase have to be changed to satisfy the energy gain per turn shown in eq.(8) in HNJ acceleration. A couple of schemes have been proposed to change the rf voltage or phase during acceleration by Ruggiero in his original paper[2], however, practical difficulties arise for realizing them. Moreover, in HNJ acceleration of medium energy heavy particle, the energy change per each turn is so large, as shown in **Fig. 1**, that adiabatic condition in longitudinal focusing (synchrotron oscillation) may not be satisfied enough to keep within the large longitudinal beam acceptance.

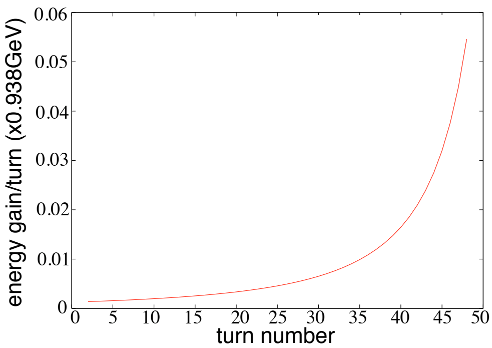


Fig. 3 Energy gain per turn as a function of turn number.

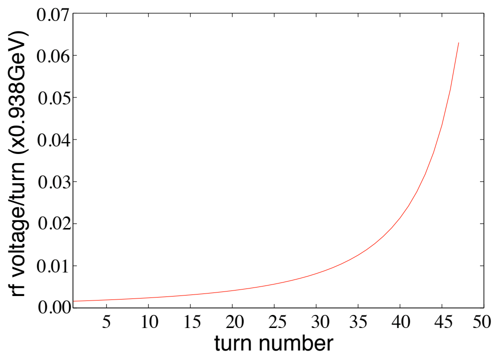


Fig.4 Required rf voltage as a function of turn number when the stable phase is 60°.

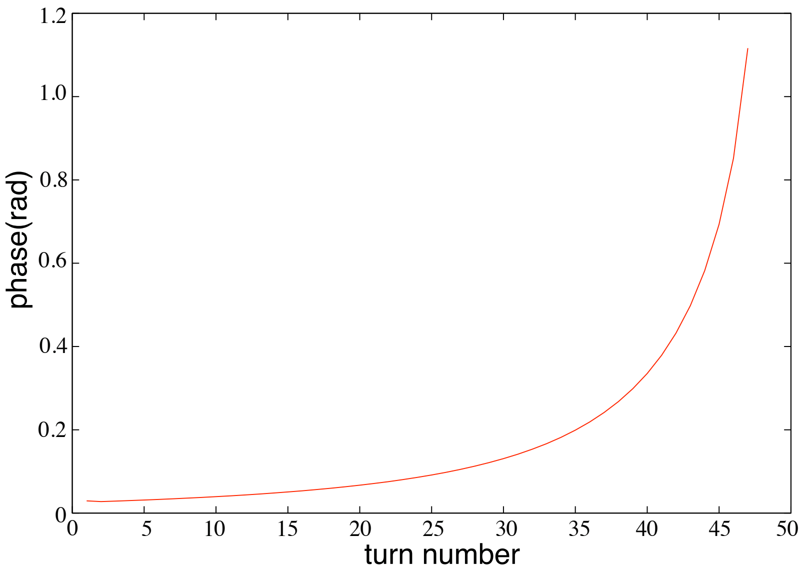


Fig.5 Stable rf phase as a function of turn number when the rf voltage is 50MV.

Figure 6 shows the particle distribution of each turn about the ring in the longitudinal phase space simulated by multi-particle beam tracking for different initial beam distributions. The beam parameters used in the beam tracking are also those shown in **Table 1** and Φ_i of eq.(17) stays constant (Φ_s=60°) and a single rf cavity located locally in the ring is assumed for beam tracking. The particle distribution in the longitudinal phase space at injection is gaussian for phase direction and zero energy spread is also assumed. Two cases for initial beam distribution were simulated; one is for the phase spread of σ=0.02rad and the other of σ=0.1rad.

As can be clearly seen from this figure, although the HNJ acceleration works in principle, the beam acceptance in longitudinal direction is very small in this case. All of the particles are accelerated up to the final energy of 500MeV when σ=0.02rad. On the other hand, a large number of particles are lost when σ=0.1rad. Without overcoming these difficulties, the HNJ acceleration may not be practical.

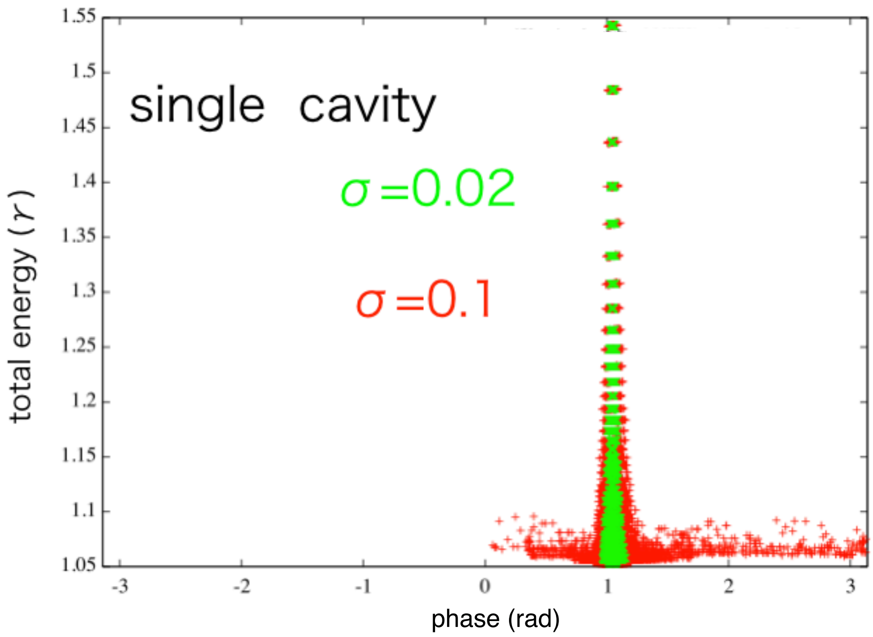


Fig. 6 Particle distribution of each turn about the ring in the longitudinal phase space simulated by multi-particle beam tracking for different initial beam distributions.

2.3 Adiabaticity in HNJ acceleration

As mentioned above, the energy gain per turn in HNJ acceleration changes largely as a function of turn number as shown in **Fig.3**. Thus, preserving the adiabatic condition of synchrotron oscillation during acceleration is important to keep a large phase space acceptance.

The criterion of adiabaticity for rf acceleration can be expressed as[7],

$$\Omega_s \gg \frac{1}{\Omega_s}\frac{d\Omega_s}{dt} , \qquad (15)$$

where $\Omega_S/2\pi$ is a synchrotron frequency. When this condition is satisfied, the particles are well trapped by a rf bucket and accelerated.

The condition of eq.(15) can be evaluated with the adiabatic parameter which is given by the following equation when the rf phase is constant (Φ_i=Φ_s)[8),9)].

$$n_{ad} = \frac{\Omega_s T_r}{1-\sqrt{V_i/\left(V_i+\Delta V\right)}} . \qquad (16)$$

Here, V_i is the total rf voltage of i-th turn and ΔV is the increment of rf voltage derived by the rf cavity after i-th turn, T_r is a transit time of the rf cavity gap. The synchrotron frequency can be given by,

$$\Omega_s = \omega\sqrt{\frac{h\eta\cos\phi_s}{2\pi\beta^2\gamma}\frac{eV}{m_0c^2}\frac{Q}{A}}, \tag{17}$$

where $\omega/2\pi$ is a revolution frequency of the particle moving around the ring, ϕ_S is a synchronous phase, m_0 is a rest mass of a particle, V is a rf voltage of a cavity, β is a fraction of the speed of light, Q/A is a charge to mass ratio.

The parameter, n_{ad}, counts the adiabaticity of the system showing how slow is the change of the bucket height with respect to the synchrotron frequency. When n_{ad}>>1, the system can be adiabatic. In the present case given by the parameters of **Table 1**, the adiabatic parameter becomes roughly, n_{ad} ~2, so that adiabaticity may not be perfectly fulfilled. Thus, the longitudinal acceptance becomes relatively small as described above.

This problem could be overcome by distributing the multi rf cavities in the ring and tuning the frequency of each rf cavity[3),4)]. If the rf system consists of *N* rf cavities, the adiabatic parameter shown in eq.(16) becomes approximately *N* times bigger than that for a single rf cavity. If *N*=16 in the present case described above, n_{ad} becomes more than 16.

The rf frequency of each rf cavity distributed homogeneously around the ring can be obtained with the following equation[3)].

$$f_{i,j} = f_{ref}\cdot\left[1+\frac{2j+N+1}{2N}\frac{\Delta_i h}{h_i}\right]^{-1}, \tag{18}$$

where *i* is the turn number, j is the cavity number and f_{ref} is a reference rf frequency.

As long as h_i is much larger than its variation $\Delta_i h$, h_0 is also much larger than $\Delta_i h$. The frequency of each cavity is independent of the turn number and is approximately given as,

$$f_j \simeq f_{ref}\left[1-\frac{\Delta_i h}{h_0}\cdot\left(\frac{2j+1}{2N}+\frac{1}{2}\right)\right]. \tag{19}$$

Thus, the rf frequency of each cavity is independent of the turn number and increases monotonically when $\Delta_i h$ is negative as a function of the cavity number. Moreover, if h_0>>$\Delta_i h$, then, f_j~f_{ref}. **Figure 7** shows a result of longitudinal beam tracking when the 16 rf cavities instead of a single rf cavity are distributed homogeneously around the ring in the case of **Table 1**.

As can be seen clearly from this figure, all of the particles are captured and accelerated up to the maximum energy even if the rms phase spread, σ(rad), of initial particle distribution is 0.4rad, which is 20 times larger than that when a single rf cavity is placed locally in the ring. Thus, the adiabatic condition in the longitudinal beam motion brought by the rf cavities distributed around the ring is important to keep a large longitudinal acceptance in HNJ acceleration.

Keeping the longitudinal adiabaticity could bring another important and valuable feature in HNJ acceleration. The rf voltage and/or phase in HNJ acceleration should vary for each beam turn to keep the criterion of the energy gain per turn shown in eq.(8). Practically, it is difficult to change the rf voltage or phase of the rf cavity during beam acceleration as mentioned before.

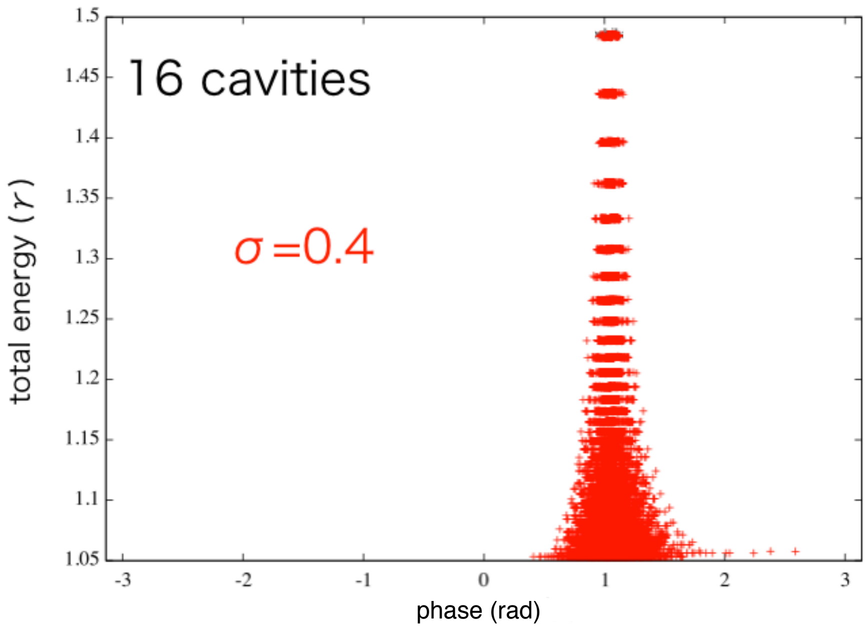


Fig.7 Result of longitudinal beam tracking when the 16 rf cavities instead of a single rf cavity are distributed homogeneously around the ring in the case of Table 1.

Normally, the response time for varying the voltage or phase in a rf cavity with high quality factor is relatively slow compared with acceleration ramping time. Even if it is realized by a rf cavity with low-quality factor, only pulsed but no cw beam operation is possible, which sacrifices operational beam duty factor and reduces the average beam current largely. However, if the longitudinal adiabatic condition of eq.(15) is well satisfied with many rf cavities distributed around the ring, this problem could be overcome.

As shown in **Fig. 8**, when the rf voltage is constant, the rf phase in HNJ acceleration can be varied during beam acceleration. If the longitudinal adiabatic condition expressed by eq.(18) is satisfied during beam acceleration, the particles could be well captured by the rf bucket and accelerated around the stable phase.

The beam tracking simulation results for the longitudinal beam motions for different phase spread initial beams are presented in **Fig. 9**. In this case, the number of rf cavities is 32 which are homogeneously distributed around the ring, and the rf voltage of 1.41MV is constant during the beam acceleration. As can be seen from this figure, the particles are well captured and accelerated up to the maximum energy following the rf stable phase, and the phase acceptance at the beam injection is quite large, which is more than 70% of 2π. This means that an adiabatic beam capture process is fulfilled in the HNJ acceleration using many rf cavities with a small rf voltage which are distributed around the ring. **Figure 10** shows the beam tracking simulation results for the longitudinal beam motions from 0 to 3turns for the phase spread of σ=2rad at initial beams. The particles are captured adiabatically and well accelerated in a bucket with harmonic number jump.

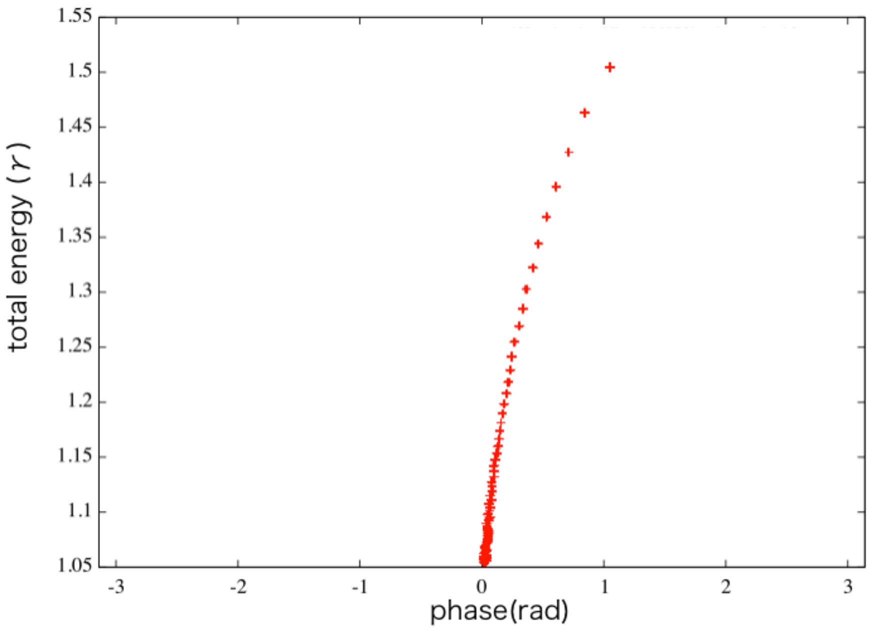


Fig.8 Variation of rf phase in HNJ acceleration during beam acceleration when the rf voltage is constant.

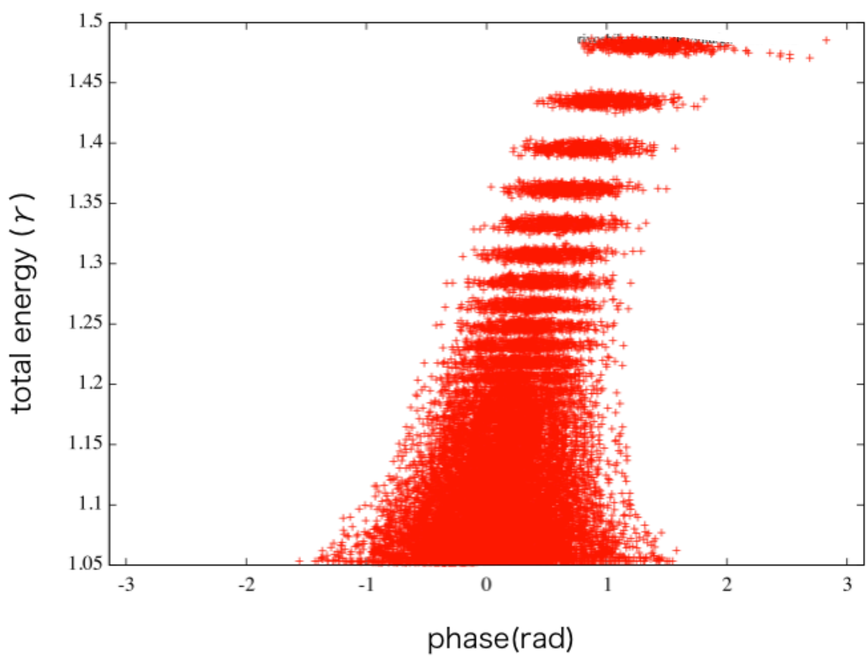


Fig.9 The beam tracking simulation results for the longitudinal beam motions for the phase spread of σ=1rad at initial beams.

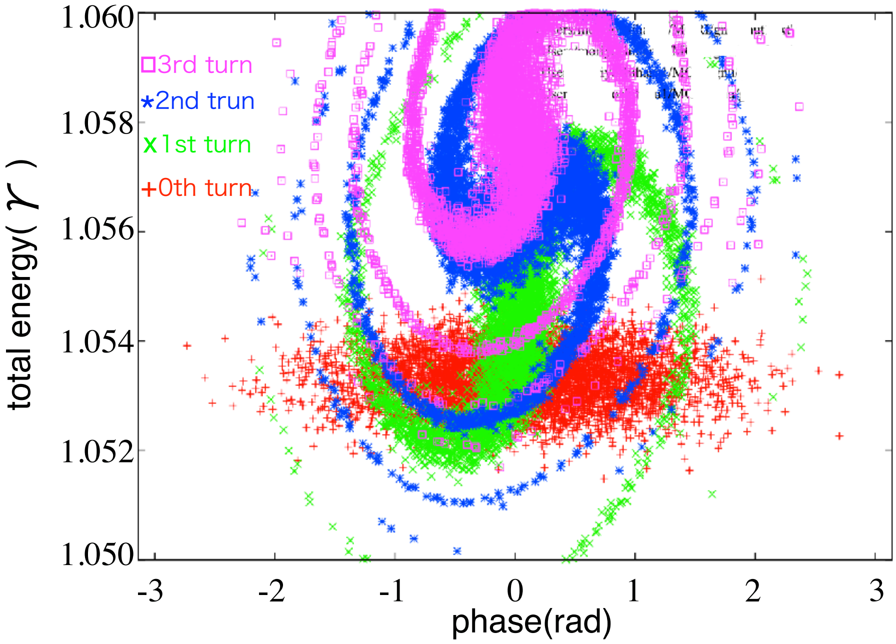


Fig.10 The beam tracking simulation results for the longitudinal beam motions from 0 to 3turns. (red: 0th turn, green: 1st turn, blue: 2nd turn, magenta: 3rd turn)

3. Summary

The vertical scaling FFAG accelerator with a harmonic number jump acceleration (HNJ) scheme, named "Harmonictron", is proposed for medium energy heavy particle acceleration in cw mode operation. Harmonictron has a couple of unique features. Since no transition energy exists in Harmonictron, a wide range of beam energy becomes possible with a monotonic change of harmonic number in HNJ acceleration. This is rather difficult in ordinary strong (AG) focusing ring accelerators without designing a special beam optics configuration such as negative momentum compaction lattice.

By distributing many rf cavities in the ring homogeneously, the rf voltage of each being relatively small, adiabatic beam capture and acceleration becomes possible and large longitudinal acceptance can be realized. By keeping enough adiabaticity in longitudinal motions for capture and acceleration of beam by distributing many rf cavities around the ring, HNJ acceleration with a constant rf voltage becomes possible, so that the cw operation with large longitudinal acceptance can be realized. This is a breakthrough for HNJ acceleration. It has been thought that either rf voltage or phase should be changed to match the energy gain criterion of integer harmonic number jump during acceleration. Moreover, the beam orbit displacement (turn separation) at the maximum energy is fairly large which could make beam extraction easy for cw beam operation.

Harmonictron is a unique accelerator to realize cw operation and essentially high intensity beam acceleration for non-relativistic heavy particles.

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References

1) Y. Mori; Advancement of the scaling FFAG accelerators for ADSR, pulsed-SNS and muon acceleration, Int. J. Mod. Phys., Vol.A26, pp.1744-1752 (2011).
2) A.G. Ruggiero; rf acceleration with harmonic number jump, Phys. Rev. ST Accel. Beams, Vol. 9, 100101 (2006).
3) T. Planche et al.; Harmonic number jump acceleration of muon beams in zero-chromatic FFAG rings, Nucl. Instr. Meth. Vol.A632, pp.7-17 (2011).
4) J.S. Berg; Harmonic number jump in a ring with cavities distributed everywhere, Proc. 2006 International Workshop on FFAG Accelerators, pp.69-76 (2007).
5) T. Ohkawa; FFAG electron cyclotron, Phys. Rev., 100, p.1247 (1955).
6) S. Brooks; Vertical orbit excursion fixed field alternating gradient accelerators, Phys. Rev. ST Accel. Beams, Vol.16, 084001 (2013).
7) B.W. Montague; RF Acceleration, CERN 77-13, pp.63-81 (1977).
8) C.G. Lilliequist and K.R. Symon; Deviations from adiabatic behavior during capture of particles into an rf bucket, MURA-491 (1959).
9) K.Y. Ng; Adiabatic capture and debunching, FNAL Report, FERMILAB-FN-0943-APC (2012).